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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 12: solving investment, percent mixture, work, and motion problems; and solving fractional equations. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

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PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

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VOLUME

12

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

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P R O G R A M M E D M A T H C O N T I N U U M

LEVEL ONE

A L G E B R A

VOLUME 12

New York Institute of Technology

Old Westbury - New York

PREFACE

This volume is one of a set of 18
that form a complete course
in
ALGEBRA - LEVEL ONE

The volume has been structured
in a multiple choice question-answer format,
with the pagination scrambled
and
is to be used in conjunction with
a program control console
utilizing
punch card input.

It is one exhibit in the demonstration of a model
developed under the direction of
the U.S. Department of Health Education and Welfare
Project 8-0157

at the

New York Institute of Technology
Westbury, New York

VOLUME 12

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IN THE STUDY GUIDE:

QUESTION;	SEGMENT:	IS ON PAGE:
1	1	$\frac{1}{1}$
1	2	$\frac{44}{1}$
1	3	$\frac{82}{1}$
1	4	$\frac{122}{1}$
1	5	$\frac{160}{1}$

C

VOLUME 12

This volume covers the following material as shown in the excerpt from the Syllabus:

SEGMENT	DESCRIPTION	(CORE) DOLCIANI	(REMEDIAL) DRESSLER	(ENRICHMENT) DODES
1	Solution of investment problems	8-13	10-8	7-2
2	Solution of percent mixture problems	8-14	10-7	
3	Solution of: fractional equations	8-15	13-4	5-3
4	Solution of work problems	8-16	13-9	7-7
5	Solution of motion problems	8-17	13-8	7-4, 7-5

READING ASSIGNMENT

VOLUME 12

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1	308	309
2	310	311
3	312	313
4	314	315
5	316	317

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

HOMEWORK ASSIGNMENT

VOLUME NO. 12

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	309	1,2,3	12110
2	309	5,6,7	12110
3	309	9,11,13	12110
4	309-310	14,15,17	12110
5	311	2,4	12210
6	311	7,8	12210
7	311	9,10	12210
8	311	12,13	12210
9	313	1,3,5	12310
10	313	7,10,13	12310
11	313	19,22	12310
12	314	23,24	12310
13	315	1,3	12410
14	315-316	4,5	12410
15	316	10,11	12410
16	316	15	12410
17	318	1,3	12510
18	318	5,6	12510
19	318	7,8	12510
20	318	11,12	12510

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is <u>not correct</u> you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is <u>correct</u> then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

Volume 12 Segment 1 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS:	48	and 50	1 1	(Sequence Number)
	54	and 56	0 4	(Type of Punch Card)
	60	and 62	1 2	(Volume Number)
	66	and 68	0 1	(Segment Number)

Your READING ASSIGNMENT for this Segment is page 308 .

Supplementary Notes

This segment investigates the techniques of solving investment problems. Investment problems involve three basic items:

First: the number of dollars invested, or principal.

Second: the interest rate, which is always given in percent form based on a full year.

Third: the amount of interest, which is found under many names; including income, return and yield.

Time must always be considered, but in the fundamental problems, we usually find that we are dealing with one full year. Since multiplying by 1 has no effect, we can ignore the time in such cases. Perhaps the simplest way to take care of the time is to attach it to the interest rate. Thus, if we are dealing with 6% interest for $\frac{1}{2}$ year, we use 3% in our calculations

$$(3\% = \frac{1}{2} \text{ of } 6\%)$$

Please turn to page $\frac{2}{1}$.

$\frac{2}{1}$

It is extremely important to understand that if $1000 is invested at 6% interest, then it will pay $60 interest at the end of a year and will still leave the original investment of $1000. You might find the same facts stated in this way: If a man invests $1000 at 6% interest, he will have $1060 at the end of a year. The technical name for the $1060 is the Amount.

The facts in an investment problem can be arranged in the following form.

	Number of dollars invested	x	Interest rate per year	x	Number of years	=	Number of dollars of interest
1st investment							
2nd investment							

Note: Later, this can be abbreviated to

Principal x Rate x Time = Interest (per year)

You will now be asked a series of questions to draw your attention to the more important points.

.....

$\frac{2}{2}$

Question 1

If Mr. K invests $600 at $5\frac{1}{2}\%$ perform the calculation to find his yearly income.

Which choice is correct?

(A) $633

(B) $30

(C) $33

(D) $55

$\frac{3}{1}$

Since the interest equals principal times rate (in one year), it should follow that rate equals interest divided by principal. In formula form:

$$I = P \times R \qquad \propto \div P$$

then $$\frac{I}{P} = R$$

If you follow this formula, you will find that this choice is not correct.

Please return to page $\frac{36}{2}$ and try question 5 again.

.....

$\frac{3}{2}$

Remember, there are two distinct parts to any verbal problem,

I: THE VARIABLES
II: THE RELATIONSHIP

Following this format, we get:

Set up the chart

I	P	R	=	I
	P	.05		.05P
	(P + 2000)	.06		.06(P + 2000)

II The total income is \$670 .

Applying the facts we have given, we get the equation:

$$.05P + .06(P + 2000) = 670$$

Please return to page $\frac{23}{1}$ and try question 11 again.

$\frac{4}{1}$

You should see clearly that there is a fraction in this choice.

The question asked for a decimal value; this is a combination of fraction and decimal, and is, therefore, not correct.

Please return to page $\frac{25}{2}$ and try question 3 again.

.....

$\frac{4}{2}$

If he invested \$5000 at 4% , that left \$5000 more to invest at 6% . Now 4% of \$5000 is \$200 and 6% of \$5000 is \$300 , which gives us a total of \$500 income.

	P	R	= I
I	5000	.04	200
II	5000	.06	300
TOTAL:			500

Since this disagrees with the problem, this choice is not correct.

Please return to page $\frac{18}{2}$ and try question 9 again.

$\frac{5}{1}$

Did you confuse income with amount? Income is the interest which is earned on the principal. Amount is the total of principal and interest.

I. Principal · Rate = Interest (Income)

II. Principal + Interest = Amount

Since he invested $600, it is unreasonable to expect that he will earn more than $600; that is, more than a 100% return.

Please return to page $\frac{2}{2}$ and try question 1 again.

$\frac{5}{2}$

According to this choice, the amount invested at 10% is more than the total amount invested. This is not correct.

Please return to page $\frac{17}{2}$ and try question 6 again.

$\frac{6}{1}$

You have done a very good job of arithmetic, but not such a good job of reading. What were you asked to find? Perhaps you should reread the notes at the beginning of this segment.

Please return to page $\frac{30}{2}$ and try question 4 again.

$\frac{6}{2}$

There are many ways to handle the decimal. Let us consider just one way. Since the decimal has two places, we can get rid of it by multiplying by 100 on both sides of the equation. Did you do that? Your mistake is probably due to difficulty with the decimal; in any case, this choice is not correct.

Please return to page $\frac{32}{2}$ and try question 8 again.

Set up the data in this form:

THE VARIABLES	Number of dollars invested	x	Interest Rate	=	Number of Dollars Interest
I:	8000		.10		
II:	x		.06		
	(8000 + x)		.07		

THE RELATIONSHIP: The interest received on investments I and II must be the same as the interest that would be gained if the total principal was invested at a 7% rate.

In order to average 7% Mrs. B has to earn 7% on the total amount she has invested. If x is the amount to be invested at 6% we have the equation:

$$.10(8000) + .06x = .07(8000 + x)$$

Solving this equation will give you the correct answer; this choice is not correct.

Please return to page 35/2 and try question 10 again.

Letting x represent the number of dollars in the share of the younger son, we find that (10,000 - x) is the share of the older. Then the income of the younger is .06x while the income of the older is

$$.09(10,000 - x)$$

Now you were told that one income is $75 more than the other. Since the larger quantity equals the smaller plus the difference, you should be able to write the equation and solve it. This choice is not correct.

Please return to page 37/2 and try question 14 again.

$\frac{8}{1}$

That is very close, but you left something out.

Did you skip some of the words in the problem?

Read it more carefully this time.

Please return to page $\frac{20}{2}$ and try question 2 again.

$\frac{8}{2}$

This choice looks reasonable, until you notice the $550 .

At 50% you would earn $500 on an investment of $1000 .

There is something wrong here.

Please return to page $\frac{21}{2}$ and try question 7 again.

You are on the right track, but there seems to be a problem with decimals and percents. Let us consider a similar problem: Find the interest rate if \$300 earns \$7.50 in a year. Since rate equals interest divided by principal and since 7.50 equals 7.5 we have:

$$R = \frac{I}{P}$$

$$R = \frac{7.5}{300}$$ We divide numerator and denominator by 3.

$$R = \frac{2.5}{100}$$ But this is a percent written in fraction form.

$$R = 2.5\%$$

Please return to page $\frac{36}{2}$ and try question 5 again.

P	R	I
$\frac{x}{3}$	.04	
$\frac{2x}{3}$	.07	

The total income is \$540 .

I: THE DATA

If we let x represent the total amount in the fund, then the part invested at 4% is $\frac{x}{3}$ and the part at 7% is $\frac{2x}{3}$.

II: THE RELATIONSHIP:

Applying our principles of investment problems, we have the equation:

$$.04(\frac{x}{3}) + .07(\frac{2x}{3}) = 540$$

This isn't the easiest equation to handle, so let's try a couple of steps together. Performing the multiplications indicated first, we get:

$$\frac{.04x}{3} + \frac{.14x}{3} = 540$$

Now we multiply both sides by 3 .

$$.04x + .14x = 1620$$

You should be able to complete the solution correctly now, and find the correct choice.

Please return to page $\frac{28}{2}$ and try question 13 again.

$\frac{10}{1}$

It appears that you had some trouble in arithmetic. Did you merely omit $\frac{1}{2}\%$ because you weren't sure how to handle it?

5% can be written as .05 or as .050

and 6% can be written as .06 or as .060

Now if 5% is .050 and 6% is .060 , it should be clear that $5\frac{1}{2}\%$ is .055 . You should be able to handle it now.

Please return to page $\frac{2}{2}$ and try question 1 again.

$\frac{10}{1}$

Since the total investment is \$12,000 the sum of the two amounts should be 12,000 .

But the total of x and x - 12,000 is

$$2x - 12{,}000$$

Therefore, this choice is not correct.

Please return to page $\frac{17}{2}$ and try question 6 again.

Since this decimal has two places, it is in hundredths; that is,

$$.35 = \frac{35}{100}$$

But this is equivalent to 35% not $3\frac{1}{2}\%$.

Note that $3\frac{1}{2}\% = .035$

Please return to page 25/2 and try question 3 again.

.....

If he invested \$4000 at 4% , that left \$6000 to invest at 6% .
Now 4% of \$4000 is \$160 and 6% of \$6000 is \$360 .

	P	x	R	=	I
I	4000		.04		160
II	6000		.06		360
TOTAL					520

Then the total interest is \$520 , which does not agree with the problem.
This choice, therefore, is not correct!

Please return to page 18/2 and try question 9 again.

12/1

Good. Let us review the techniques for solving the problem:

Our solution can be put in this form:

I THE VARIABLES:

Number of dollars invested	·	Interest rate	=	Number of dollars interest
P		.05		.05P
P + 2000		.06		.06(P + 2000)

II THE RELATIONSHIP: The total yearly income is $670 .

Then our equation is:

$$.05P + .06(P + 2000) = 670 \quad [D$$
$$.05P + .06P + 120 = 670 \quad [\text{Collect}$$
$$.11P + 120 = 670 \quad \propto -120$$
$$.11P = 550 \quad \propto (100)$$
$$11P = 55000 \quad \propto \div 11$$
$$P = 5000$$

III CHECK

P	x	R	=	I
5000		.05		250
7000		.06		420
				670

Then this choice is correct.

Please proceed to question 12 below.

.....

12/2

Question 12

In a year, one sum of money invested at 6% yields the same as another investment at 8% . If the total investment is $7000 apply your knowledge to find the set containing the amount invested at 8% .

(A) {$12000, $1600, $2000}
(B) {$1500, $2500, $3000}
(C) {$2400, $3600, $4800}
(D) {$2800, $3200, $3600}

If you invest a sum of money, you expect to earn interest which is usually very much less than the investment. If you invest $30 and earn $30 interest in one year, you are getting a 100% return on your money! This is not a normal rate of interest.

Please do not confuse the interest you earn with the amount that you have at the end of the year. The amount is made up of the interest and the original investment.

Please return to page $\frac{20}{2}$ and try question 2 again.

This choice implies that the total interest rate is $8\frac{1}{2}\%$.
But there is no such thing as a total interest rate. If you earn 100% on one investment, and 100% on a second investment, do you earn 200% on the total? Of course not; you are earning 100% on the total. Remember you cannot add percentages; you can only add amounts.

Please return to page $\frac{21}{2}$ and try question 7 again.

$\frac{14}{1}$

You have the right idea, but you have made an error in arithmetic.

How much was the interest he earned in the year?

Please return to page $\frac{30}{2}$ and try question 4 again.

$\frac{14}{2}$

Remember that if one income exceeds the other, it is larger than the other. Then we can still use the principle: the larger quantity equals the smaller plus the difference.

If g represents Mr. G's investment, then

$$g + 4000$$

is Mr. H's investment.

Their incomes, in order, are:

$$.10g \text{ and } .05(g + 4000)$$

Applying these expressions to the principle stated above, we get an equation whose solution is not in the set you chose.

Please return to page $\frac{24}{2}$ and try question 15 again.

$\frac{15}{1}$

When the time is one year, interest equals principal multiplied by rate.

Did you multiply \$600 by the $5\frac{1}{2}\%$ written in decimal form?

This choice is not correct.

Please return to page $\frac{2}{2}$ and try question 1 again.

$\frac{15}{2}$

It is true that the second investment earns twice the interest rate of the first. But you were not told that the second investment was twice as large. How can you make use of the fact which was given.

Please return to page $\frac{17}{2}$ and try question 6 again.

16/1

It is true that $3\frac{1}{2}$ is equal to 3.5 but you were dealing with percent. Consider this example: 5% is equal to .05 not 5 .

Apply this idea to your problem.

Please return to page 25/2 and try question 3 again.

16/2

When something is added to 20 ounces, the result is no longer 20 ounces. Then this choice is not correct.

Place the given data in chart form.

Remember there is the original solution, the added solution, and the final mixture; a total of three rows of information.

Please return to page 50/2 and try question 2 again.

Since interest equals principal times rate (for one year) it follows that rate equals interest divided by principal.

In formula form, $I = P R$ $\div P$

then $\frac{I}{P} = R$

Using this formula, we have:

$R = \frac{25}{500}$ [Divide numerator and denominator by 5.

$R = \frac{5}{100}$ [But, this is a percent in fraction form.

$R = 5\%$

Then this choice is correct.

Please proceed to question 6 below.

Question 6

A man invests part of $12,000 at 5% and the balance at 10% .

If x represents the amount invested at 5% choose the correct representation of the amount invested at 10% .

(A) $12,000 + x$

(B) $x - 12,000$

(C) $12,000 - x$

(D) $2x$

18
1

There are many ways to handle the decimal. Let us consider just one way

Since the decimal has two places we can get rid of it by multiplying by 100 As you should have learned, multiplying by 100 has the effect of moving the decimal point two places to the right. Then we have:

Given:	x	$+ .04x$	$=$	520	$\times(100)$
	$100x$	$+ 4x$	$=$	52000	[Combine terms
		$104x$	$=$	52000	$\div 104$
		x	$=$	500	

Then this choice is correct.

Please proceed to question 9 below.

.....

18
2

Question 9

A man invested part of his inheritance of $10,000 at 4% and the remainder at 6% . Check the following choices to find the amount invested at 4% if his annual income is $480 .

(A) $6000

(B) $5000

(C) $4000

(D) $3000

Perhaps you ought to take another look at your arithmetic since there seems to be an error there.
What were you asked to find? It appears that you were looking for the wrong quantity.

Please return to page 30/2 and try question 4 again.

.....

Did you put the given data into chart form?

I THE VARIABLES:

If we let n stand for the number of dollars invested at 8% then $(7000 - n)$ will be the number of dollars invested at 6% .

P	x	R	=	I
n		.08		$.08n$
$7000 - n$		.06		$.06(7000 - n)$

II THE RELATIONSHIP:

But the two yields, or incomes, are the same. Thus we have the equation:

$$.08n = .06(7000 - n)$$

Solving this will give us the value of n . However, we will find that the set in this choice does not contain the correct value of n

Please return to page 12/2 and try question 12 again.

$\frac{20}{1}$

	Principal Invested	x	Rate Per Year	x	Number of Years	=	Interest
Formula (1)	P	x	R	x	T	=	I

When the time is one year, interest equals principal multiplied by rate.

Formula (2)	P	x	R			=	I

Putting $5\frac{1}{2}\%$ in decimal form, we get .055

P	x	R	=	I
600		.055		33

Then we multiply $600 by .055 getting $33 .

Then this choice is correct.

Please proceed to question 2 below.

$\frac{20}{2}$

Question 2

Perform the calculation to find how much Miss R will earn in two years if she invests $250 at 4% .

(A) $20

(B) $10

(C) $270

(D) $260

$\frac{21}{1}$

If the sum of two quantities is given, then subtracting one quantity from the sum gives the other. Therefore, if the total is 12,000 and one part is x the other is (12,000 - x) .

Then this choice is correct.

Please proceed to question 7 below.

.....

$\frac{21}{2}$

Question 7

If \$1000 is invested at $5\frac{1}{2}\%$ and p dollars is invested 3% , apply your knowledge to find the number of dollars in total yearly income.

(A) (550 + .03p)

(B) (55 + .03p)

(C) .085(1000 + p)

(D) None of these

Good, let's review the problem.

I THE VARIABLES:

Let x represent the amount to be invested at 6% .

Then we have:

Number of Dollars Invested	• Interest Rate	= Number of Dollars Interest
8000	.10	800
x	.06	.06x
8000 + x	.07	560 + .07x

II THE RELATIONSHIP:

The total interest received from the two investments is equal to the interest that would have been received if the entire principal were invested at 7% .

Therefore, our equation is:

$$800 + .06x = 560 + .07x \quad \text{<- } 560 - .06x$$

$$240 = .01x \quad \text{<÷ } .01$$

$$24000 = x$$

III CHECK

P	• R	= I
8000	.10	800
24000	.06	1440
32000	.07	2240

Then this choice is correct.

Please proceed to question 11 on page 23/1 .

Question 11

One sum of money is invested at 5% and another, ($2000 larger than the first), is invested at 6%. If the total yearly income is $670 apply your knowledge to find P; the amount of money invested at 5%.

Which is the correct statement about the value of P ?

(A) $P < \$5500$

(B) $\$5500 < P < \$6500$

(C) $\$6500 < P < \$7500$

(D) $P > \$7500$

.....

The basic principle here is that Amount equals Principal plus Interest.

Then, using x for the principal, we get the equation:

$$x + .06x = 9010$$

If you solve this equation correctly, you will find that the solution is not in the set you have chosen.

Please return to page $\frac{34}{2}$ and try question 16 again.

24/1

I THE VARIABLES:

Let x represent the share of the younger son. We have:

	P	•	R	=	I
Younger	x		.06		.06x
Older	(10,000 - x)		.09		.09(10,000 - x)

and the equation is:

$$.06x = .09(10{,}000 - x) + 75 \qquad [D$$
$$.06x = 900 - .09x + 75 \qquad \swarrow + .09x \text{ and combine}$$
$$.15x = 975 \qquad \swarrow (100)$$
$$15x = 97500 \qquad \swarrow \div 15$$
$$x = 6500$$

II THE RELATIONSHIP:

The new feature in this problem is the fact that the one income is larger than the other. We can use the principle that the larger quantity equals the smaller plus the difference.

III CHECK:

P	•	R	=	I
6500		.06		390
3500		.09		315
Difference:				75

Then this choice is correct.

Please proceed to question 15 below.

.....

24/2

Question 15

Mr. G invests some money at 10% while Mr. H invests $4,000 more at 5%. At the end of a year, Mr. G's income exceeds Mr. H's income by $50. Apply your knowledge to find the set containing the amount of money Mr. G invested.

(A) {$2000, $2500, $3000} (B) {$3500, $4000, $4500}

(C) {$5000, $5500, $6000} (D) None of these.

Very good. If you use the basic formula that interest equals the product of principal, interest rate, and time, you get

Principal	x	Rate of Interest	x	Number of Years	=	Interest
250	x	.04	x	2	=	20

$250 \times (.04 \times 2) =$

$250 \times .08 =$ (This is an application of the ASSOCIATIVE PROPERTY)

Since the time is two years, we may figure that Miss R is earning twice 4% or 8% on the $250 .

Please proceed to question 3 below.

Question 3

Which do you recognize as the decimal equivalent of $3\frac{1}{2}\%$?

(A) $.03\frac{1}{2}$

(B) .35

(C) 3.5

(D) .035

$\frac{26}{1}$

You may have worked too rapidly; you have an error. One of the other choices does have the correct answer.

Please return to page $\frac{21}{2}$ and try question 7 again.

$\frac{26}{2}$

Since there are 24 quarts of solution which contain 4 quarts of antifreeze, this choice is not correct.

Arrange your data in chart form. The final line (the mixture) should appear like this:

Number of Units of Solution	x	Percent Antifreeze	=	Number of Units of Antifreeze
24		x		4

Now write the equation relating these values and solve for x .

Convert x to a percent.

Please return to page $\frac{45}{2}$ and try question 1 again.

$\frac{27}{1}$

If he invested $3000 at 4% , that left $7000 to invest at 6% .
Now 4% of $3000 is $120 and 6% of $7000 is $420 .

	P	x	R	=	I
I	3000		.04		120
II	7000		.06		420
TOTAL					540

Then the total interest is $540 which does not agree with the problem.
Then this choice is not correct.

Please return to page $\frac{18}{2}$ and try question 9 again.

$\frac{27}{2}$

When two solutions are mixed together, the total amount is the sum of the two separate amounts. Since this choice is only one of the two numbers that should be added together, it is not the correct choice.

Please return to page $\frac{62}{2}$ and try question 4 again.

$\frac{28}{1}$

Good. Let's outline the procedure:

I THE VARIABLES:

We can set up the problem in this way, using n for the number of dollars invested at 8% :

Number of dollars invested	·	Interest rate	=	Number of dollars interest
n		.08		.08n
7000 - n		.06		.06(7000 - n)

II THE RELATIONSHIP:

Since the two incomes are equal, we have:

.08n	=	.06(7000 - n)	[D
.08n	=	420 - .06n	∝ + .06n
.14n	=	420	∝ (100)
14n	=	42000	∝ ÷ 14
n	=	3000	

III CHECK

P	·	R	=	I
3000		.08		240
4000		.06		240

This choice contains this value; it is, therefore, correct.

Please proceed to question 13 below.

.....

$\frac{28}{2}$

Question 13

One-third of a fund is invested at 4% and the rest is invested at 7% . If the total annual income is $540 apply your knowledge to find the set containing the amount invested at 4% .

(A) {$5000, $7000, $9000} (B) {$4500, $6000, $7500}

(C) {$1800, $2200, $2600} (D) {$1000, $2000, $3000}

You have made a mistake somewhere, since one of the other choices is correct. Were you confused by the word exceeds? It has the same meaning as is larger than. Using the principle you have already met, we can write: G's income equals H's income plus 50 . You should be able to continue from there.

Please return to page 24/2 and try question 15 again.

Since the original alloy was 2% tungsten, less than 50 pounds was tungsten. Then adding 5 pounds to it cannot result in a total of 55 pounds.

Please return to page 56/2 and try question 3 again.

$\frac{30}{1}$

$3\frac{1}{2}\%$ is equivalent to $\frac{3\frac{1}{2}}{100}$.

Since $3\frac{1}{2} = 3.5$, we can write the fraction as $\frac{3.5}{100}$.

But dividing by 100 has the effect of shifting the decimal place two places to the left in the number. Thus, we get .035 since a zero must be placed in the empty place between the decimal point and the 3 .
If this idea is strange to you, it is worth learning. Of course, if you did the division as a long division, you would still get this result since it is correct.

This tranformation can also be explained by using the technique of reducing the fraction.

$$\frac{3.5}{100} \times \frac{\frac{1}{10}}{\frac{1}{10}} = \frac{.35}{10} \times \frac{\frac{1}{10}}{\frac{1}{10}}$$

$$= \frac{.035}{1}$$

$$= .035$$

Please proceed to question 4 below.

.....

$\frac{30}{2}$

Question 4

Apply your knowledge to find how much money Mr. P will have after a year if his \$10,000 is earning 9% interest.

(A) \$900

(B) \$10,900

(C) \$19,000

(D) \$9,000

It is true that there are now 25 ounces of solution, but the amount of salt has not changed. Since the amount of salt is the same, while the amount of the total solution is changed, it follows that the percent of salt cannot be the same as it was.

Place the given data in chart form. Remember, there are three rows of facts; namely, one for the original solution, a second for the added solution, and a third for the final mixture.

Please return to page 50/2 and try question 2 again.

You have the correct answer to a question, but not this question.
You were asked to find the number of pounds of salt.

Please return to page 48/2 and try question 5 again.

$\frac{32}{1}$

In order to calculate the total income, we must find the separate figures and then add them. $5\frac{1}{2}\%$ on \$1000 is \$55 .

Writing 3% in decimal form we have .03 ; and .03 times p gives .03p . Then adding them gives the value of this choice; which is, therefore, correct.

	P	• R	= I
I	1000	.055	55
II	P	.03	.03p
		Total interest	(.55 + .03p)

Note: It is customary not to include dollar signs and other measurements in the table or in the equation.

Please proceed to question 8 below.

$\frac{32}{2}$

Question 8

Apply your knowledge to find the correct statement about the value of x which satisfies the equation

$$x + .04x = 520$$

(A) $x < 102$

(B) $102 < x < 302$

(C) $302 < x < 502$

(D) $x > 502$

$\frac{33}{1}$

Without considering the percent antifreeze in the resulting solution, let us look at the amount of solution.

We started with 20 quarts and added something to it, then it cannot still be 20 quarts.

Please return to page $\frac{45}{2}$ and try question 1 again.

$\frac{33}{2}$

This choice is not correct. If you read the question again carefully, you should find that the correct result is easy to get; and it is offered in one of the other choices.

Please return to page $\frac{62}{2}$ and try question 4 again.

I THE VARIABLES:

Remember that exceeds means is more than. Then we can use the principle: the larger quantity equals the smaller plus the difference, to write an equation.

First, let g represent Mr. G's investment. Then we have:

	P	·	R	=	I
Mr. G	g		.10		.10g
Mr. H	(g + 4000)		.05		.05(g + 4000)

III CHECK

	P	·	R	=	I
	5000		.10		500
	9000		.05		450
	Difference				50

II THE RELATIONSHIP:

Mr. G's income exceeds Mr. H's income by $50 and the equation is:

$$.10g = .05(g + 4000) + 50 \quad \text{[D}$$
$$.10g = .05g + 200 + 50 \quad \text{✓ − (.05g) and combine}$$
$$.05g = 250 \quad \text{✓ (100)}$$
$$5g = 25000 \quad \text{✓ ÷ 5}$$
$$g = 5000$$

Since the set you chose has this number, this choice is correct.

Please proceed to question 16 below.

Question 16

A man can invest his money so as to earn 6% interest, and he wishes to have $9010 at the end of one year. Apply your knowledge to find how much he should invest today.

Which set contains the correct answer?

(A) {$6600 , $6900, $7200}
(B) {$6500 , $6800, $7100}
(C) {$7400 , $7600, $7800}
(D) {$7900 , $8200, $8500}

If he invested $6000 at 4%, that left $4000 to invest at 6% .
Now 4% of $6000 is $240 and 6% of $4000 is $240 .
Then the total interest is $480 and this choice is correct.

	P	x	R	=	I
I	6000		.04		240
II	4000		.06		240
					480
To find average:	10,000		$\frac{480}{10000}$		480

It is worth noting that $480 is 4.8% on $10,000 .
This is described by saying that the man has earned an <u>average</u> of 4.8% on his investment. It is impossible to get this by calculating a simple average of any of the numbers in the problem. The "average rate" may be defined as that rate on the whole investment which would yield the same amount of total interest as the two separate investments.

Please proceed to question 10 below.

.....

Question 10

If Mrs. B can invest $8000 at 10% interest, apply your knowledge to find how much she should invest at 6% interest so that she will average 7% on her total investment.

(A) $8000
(B) $16,000
(C) $20,000
(D) $24,000

$\frac{36}{1}$

The interest on $10,000 at 9% for a year is $900 .
Then Mr. P will have his original $10,000 plus the $900 and this choice is correct.

	Principal	·	Rate per Year	=	Interest
I	$10,000		.09	=	$900

	Principal	+	Interest	=	Amount at End of Year
II	$10,000	+	900	=	10,900

Please proceed to question 5 below.

.....

$\frac{36}{2}$

Question 5

Apply your knowledge to find the interest rate if $500 earns $25 in a year.

(A) 25%

(B) .05%

(C) .5%

(D) 5%

Good. Check out this method for solving the problem.

I THE VARIABLES

If x is the amount in the fund, we proceed as follows:

P	x	R	=	I
$\frac{x}{3}$		.04		$\frac{.04x}{3}$
$\frac{2x}{3}$		.07		$\frac{.14x}{3}$

III CHECK

P	•	R	I
3000		.04	120
6000		.07	$\frac{420}{540}$

II THE RELATIONSHIP:

The total interest amounted to \$540 . Therefore,

$$\frac{.04x}{3} + \frac{.14x}{3} = 540 \quad \text{Multiply both sides by } 3$$

$$.04x + .14x = 1620 \quad \text{Combine like terms}$$

$$.18x = 1620 \quad (100)$$

$$18x = 162000 \quad \div 18$$

$$x = 9000$$

Then one-third of this is \$3000 , the amount invested at 4% .

Please proceed to question 14 below.

.....

Question 14

A man leaves an estate of \$10,000 to be divided by his two sons. The older son invests his share at 9% and the younger son invests his share at 6% . If the yearly income of the younger son is \$75 more than that of the older, apply your knowledge to find S , the younger son's share of the estate.

(A) $S > \$8000$

(B) $\$8000 > S > \$6000$

(C) $\$6000 > S > \$4000$

(D) $\$4000 > S > \$2000$

$\frac{38}{1}$

It is true that there are now 25 ounces of solution, but the amount of salt has not changed. Since we have the same amount of salt in a larger amount of solution, it follows that the salt is a <u>smaller</u> fraction of the total.

Then this choice is not correct.

Please return to page $\frac{50}{2}$ and try question 2 again.

$\frac{38}{2}$

The problem might be handled this way: Let x represent the number of gallons of 20% solution to be used. Then 50 - x represents the amount of 10% solution. Setting up the form for this problem:

Number of gallons of Solution	Percent sugar	=	Number of gallons sugar
x	20		.20x
(50 - x)	10		(5 - .10x)
50	32		16

Then adding in the third column gives the equation:

$$.20x + 5 - .10x = 16$$

Now, solving this equation you will find that this choice is not correct. Always check your answer in the original problem to see whether it makes sense.

Please return to page $\frac{67}{2}$ and try question 6 again.

I THE VARIABLES:

If we let x represent the principal, the interest is .06x and the equation is

$$x + .06x = 9010 \quad \text{«(100)}$$
$$100x + 6x = 901000 \quad \text{[Combine}$$
$$106x = 90100 \quad \text{«÷106}$$
$$x = 8500$$

III CHECK:

P	x	R	=	T
8500		.06		510

P	+	I	=	Amount
8500	+	510	=	9010

III THE RELATIONSHIP:

In this problem, $9010 is called the Amount. Remember

Amount = Principal + Interest

Then this choice is correct.

You have now completed this Segment. Hand in the PUNCH CARD.

You should have entered in your NOTEBOOK the following:

(1) Amount = Principal + Interest

(2) Number of dollars invested x interest rate = Number of dollars interest

(P · R = I)

(3) Interest, income, return, yield; all mean the same.

You should now be able to complete ASSIGNMENT 12 , problems 1 - 4 .

$\frac{40}{1}$

Does 2 percent plus 5 pounds equal 7 pounds?

Of course not.

Were there 2 pounds of tungsten in the original alloy? Avoid doing arithmetic with numbers until you know what they mean and what meaning the result will have.

Please return to page $\frac{56}{2}$ and try question 3 again.

$\frac{40}{2}$

When water is evaporated from acid solution, the water being removed has 0% acid. Then no acid has been removed and, therefore, there is some acid left.

Then this choice is not correct.

Please return to page $\frac{63}{2}$ and try question 8 again.

41/1

If x represents the number of gallons of 20% solution to be used, we get the equation

$$.20x + 5 - .10x = 16$$

Solving this, we find that

$$x = 110$$

which is certainly larger than 25 . But the total number of gallons of the mixture was to be only 50 .

Then this value is not correct.

Please return to page 67/2 and try question 6 again.

.....

41/2

Using x as the amount of 20% solution, we have:

	Number of Pounds of Solution	· Percent salt	= Number of Pounds of Salt
1st Solution	x	20	$.20x$
2nd Solution	30	5	1.5
Mixture	$x + 30$	10	$.10x + 3$

In the third column, the sum of the first two lines equals the third. If we write the equation stating this and solve it correctly, we find that this choice is not correct.

Please return to page 66/2 and try question 10 again.

$\frac{42}{1}$

If we start with 20 quarts and evaporate (remove) something from it, we cannot have 20 quarts remaining.

Please return to page $\frac{51}{2}$ and try question 7 again.

$\frac{42}{2}$

It is helpful to set-up the data in chart form:

We may proceed in this manner:

	Number of Gallons of Mouthwash	.	Percent of Ingredient X	=	Number of Gallons of Ingredient X
1.	1000		5		.05(1000)
2.	n		0		0
M.	1000 + n		2		.02(1000 + n)

Now, in the third column, the first item plus the second equals the third. If you will write the equation which states this fact and solve it correctly, you will find that this choice is not correct.

Please return to page $\frac{69}{2}$ and try question 12 again.

In general, there are two ways that you might explore in order to answer this question: one is to find the amount of salt in each solution and then add; the other is to find the total amount of solution and its percent salt and then calculate the amount of salt. You will see that you must use the second method which relates to the total mixture.

You have made a mistake in your calculation since this choice is not correct.

Please return to page 48 and try question 5 again.

Certainly 20 minus 3 equals 17 but that has nothing to do with this problem. How many pints of acid were there in the original solution, and how many were removed?

Please return to page $\frac{54}{2}$ and try question 9 again.

Volume 12 Segment 2 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS:	48 and 50	1	2	(Sequence Number)
	54 and 56	0	4	(Type of Punch Card)
	60 and 62	1	2	(Volume Number)
	66 and 68	0	2	(Segment Number)

Your READING ASSIGNMENT for this Segment is page 310 .

SUPPLEMENTARY NOTES

A solution, as you meet it in this course, is usually a mixture containing a substance dissolved in water. The substance gives its name to the solution; thus a 10% alcohol solution would mean that 10% of the mixture is alcohol and the other 90% is water or some other liquid that is specified. If there were 50 gallons of such a solution, it would contain 5 gallons of alcohol and 45 gallons of water. Some named substances are actually mixtures; two common ones are milk and cream. They contain various substances mixed with water; the one substance which is of importance commercially is called "butterfat." The richness of the mixture determines whether it is called "milk" or "cream."

You should keep in mind that water means pure water, which is 0% salt, 0% alcohol, 0% butterfat. In the same way, salt means pure salt, which is 100% salt.

You will also meet mixtures of metals called alloys. In these, you will be told the names of both substances; for example, an alloy of tin and copper contains 15% tin.

We will generally deal with a mixture of two different solutions to make a third. However, we may add a pure substance (100%) to a solution, which strengthens it. Or we may add water (0%) to a solution which weakens it. It is possible to remove water (0%) from a solution by evaporation; of course, this strengthens the solution.

Please go on to page 45 .
1

The basic formula for these percent mixture problems is

Number of Units of Solution	·	Percent Ingredient	=	Number of Units of Ingredients
First				
Second				
Total				

In the first column, adding gives the total and the same holds true for the third column; but DO NOT attempt to add the percents.

You will now be asked a series of questions to draw your attention to the more important points.

Question 12

After 4 quarts of antifreeze are added to 20 quarts of water, what is the amount of the solution and its percent antifreeze?

Perform the calculations to find the correct choice.

(A) 24 quarts of $16\frac{2}{3}\%$ antifreeze

(B) 24 quarts of 20 % antifreeze

(C) 20 quarts of 20 % antifreeze

(D) 20 quarts of 25 % antifreeze

46/1

The original alloy had some tungsten in it. Then if we add 5 pounds of tungsten, there must be more than 5 pounds in the final mixture.

Please return to page 56/2 and try question 3 again.

46/2

Before it is possible to find the percent of acid in the final solution, we must find the number of pints of acid it contains. The original solution contained 20% of 15 pints, or 3 pints, of pure acid.
The evaporation of water removed no acid so that 3 pints still remain.
But the evaporation did reduce the total amount of solution.

	N	x	R	=	I	
Solution	15		.20		.20 (15)	
Water	3		0		0	evaporated
Mixture	12		x		12x	

Dividing the number of pints of acid remaining by the number of pints of solution remaining gives the correct result. This choice is not correct.

Please return to page 63/2 and try question 8 again.

This choice is not correct. There is enough information given to determine the correct solution to this problem. However, it is important to interpret correctly the results you obtain.

Please return to page 67/2 and try question 6 again.

Let us assume that this choice is correct and check the figures which result; there would be 70 cc. of 10% solution. Then:

	Number of cc. of Solution	· Percent Salt	= Number of cc. of Salt
1.	30	5	1.5
2.	70	10	7
M.	100	7	7

But the items in the third columns have to check when added. Since 1.5 + 7 does not equal 7 this does not check. Then this choice is not correct.

Please return to page 52/2 and try question 11 again.

48
1

If two quarts are mixed with 20 quarts, the result will, of course, contain 22 quarts. You should not be fooled into trying to use any of the other numbers in the problem to get an answer. The other information in the problem has nothing to do with this question.

Please proceed to question 5 below.

48
2

Question 5

After mixing 20 pounds of 5% salt solution with 30 pounds of another salt solution, we find that we have a 6% salt solution. Apply your knowledge to find the number of pounds of salt in the final solution.

(A) 50

(B) 18

(C) 3

(D) It cannot be determined.

There was 15% acid in the original 20 pint solution, not in the final solution. Then the only figure on which you can calculate 15% sensibly is the 20 pints. Then this choice is not correct.

Please return to page $\frac{54}{2}$ and try question 9 again.

The original solution contained 20 cc. of acid out of a total of 100 cc. If the choice is correct, the 20 is 25% of the remaining 75 cc . But this is not so.

Then this choice is not correct.

Please return to page $\frac{55}{2}$ and try question 13 again.

$\frac{50}{1}$

Good. Let's arrange the given data in chart form.

	Number of Units of Solution	· Percent of Ingredient	= Number of Units of Ingredient
Water	20	0	0
Antifreeze	4	100	4
Final Mixture	24	x	4

Since: $24x = 4$ ÷ 24

$x = \frac{4}{24}$ [Reduce

$x = \frac{1}{6}$ [Convert to %

$\frac{1}{6} = .16\frac{2}{3}$

$6\overline{)1.00}$

$x = 16\frac{2}{3}\%$

Your choice was correct.

Now proceed to the next problem below.

.....

$\frac{50}{2}$

Question 2

If 5 ounces of water are added to 20 ounces of a 10% salt solution, which statement about the resulting solution do you recognize as correct?

(A) There are 20 ounces containing less than 10% salt.

(B) There are 25 ounces containing less than 10% salt.

(C) There are 25 ounces containing 10% salt.

(D) There are 25 ounces containing more than 10% salt.

If a 20% solution and 10% solution are mixed, the resulting mixture always has a percent between the two original percents. Then this choice is correct, and no further calculation is needed. Mixing a 20% solution and a 10% solution could never result in a 32% solution. If you proceed in a standard manner, letting x represent the amount of the 20% solution to be used, you get the equation

$$.20x + 5 - .10x = 16$$

Solving this, we find the value of x to be 110 . But the total solution is to be 50 gallons. Therefore, this value of x is impossible, and this choice is correct.

Please proceed to question 7 below.

.....

Question 7

When 2 quarts of water are evaporated from 20 quarts of a 15% salt solution, how many quarts of solution remain?
Which choice do you recognize as correct?

(A) 20
(B) 18
(C) 3
(D) 1

52/1

I THE VARIABLES:

Using x as the amount of the 20% solution, we have:

	Number of Pounds of Solution	Percent Salt	= Number of Pounds of Salt
1st Sol.	x	20	.20x
2nd Sol.	30	5	1.5
MIXTURE	x + 30	10	.10x + 3

II THE RELATIONSHIP:

The amount of salt in the 1st solution plus the amount of salt in the 2nd solution is equal to the number of pounds in the mixture.

III CHECK:

15	.20	3
30	.05	1.5
45	.10	4.5

Then this choice is correct.

Please proceed to question 11 below.

.....

52/2

Question 11

A druggist requires 100 cc. of a 7% salt solution, but has available only a 5% solution and a 10% solution. How much of the 5% solution should he mix with the 10% solution to produce the required result?

Apply your knowledge to find which answer checks.

(A) 30 cc. (C) 40 cc.

(B) 70 cc. (D) 60 cc.

If you reconsider this problem, you will find that there is enough information to answer the question, using one of these two ways to arrive at the figure you need:

(1) Add the amounts of salt in each separate solution.

(2) Find the total amount of the final solution and its percent salt.

(3) Choose the appropriate method.

Please return to page 48/2 and try question 5 again.

.....

Letting x be the number of pounds of salt to be used, we have:

	Number of Pounds of Solution	· Percent Salt	= Number of Pounds of Salt
1st Sol.	40	5	.05(40)
2nd Sol.	x	100	1(x)
	40 + x	24	.24(40 + x)

II: THE RELATIONSHIP

The number of pounds of salt in the first solution plus the number of pounds added equals the amount found in the final mixture. That is, in the third column, the first number plus the second equals the third. This gives us an equation which we can solve, but this set does not contain the correct solution. Check your computation.

Please return to page 61/2 and try question 14 again.

54
1

The original 15 pints contained 20% or one-fifth acid. Then there were 3 pints of acid when we started. The evaporation did not affect the amount of acid, but it did reduce the amount of solution to 12 pints. Therefore, we find that we have 3 pints of pure acid out of 12 pints of solution; and this choice is correct.

	N	x	R	=	I
Solution	15		.20		.20(15)
Water	3		0		
Mixture	12		x		12x

CHECK:

15	.20	3.0
3	0	0
12	.25	3.0

The new mixture has the same amount of acid as the original solution.

$$12x = .20(15)$$

$$12x = 3 \qquad \div 12$$

$$x = \frac{1}{4} \qquad [\text{ Convert to } \%$$

$$x = 25\%$$

Please proceed to question 9 below.

54
2

Question 9

If 3 pints of water are evaporated from 20 pints of a 15% solution of acid, how many pints of pure acid remain?

Choose the correct result.

(A) 17

(B) 3

(C) 2.05

(D) 0

We may proceed in this manner:

I THE VARIABLES

	Number of Gallons of Mouthwash	Percent of Ingredient X	=	Number of Gallons of Ingredient X
1.	1000	5		.05(1000)
2.	n	0		0
M.	(1000 + n)	2		.02(1000 + n)

II THE RELATIONSHIP:

The number of gallons of ingredient "X" in the mixture is equal to the amount in the original solution plus what was added.

Then we have:

$.02(1000 + n) = .05(1000)$ [D

$20 + .02n = 50$ ∝ − 20

$.02n = 30$ ∝(100)

$2n = 3000$ ∝ ÷ 2

$n = 1500$

CHECK:		
1000	.05	50
1500	0	0
2500	.02	50

Therefore, this choice is correct.

Please proceed to question 13 below.

Question 13

A bottle containing 100 cc. of a 20% acid solution was left open. When discovered, it was found that due to the evaporation of water, it was now a 25% solution.

Apply your knowledge to find how much solution was still in the bottle.

(A) 80 cc.

(B) 75 cc.

(C) 25 cc.

(D) 20 cc.

56
1

Let's review the procedure:

	Number of Ounces of Solution	x	Percent Salt	=	Number of Ounces of Salt
Solution	20		10	=	.10(20)
Water	5		0		0
Mixture	25		x		.10(20)

$25x = 2$ $\quad$ $\frac{x}{1} = 25$

$x = \frac{2}{25}$ $\quad$ [Convert $\frac{2}{25}$ to a %

$x = 8\%$ $\quad$ $\frac{2}{25} = \frac{(8)}{100}$

Adding 5 ounces to 20 ounces results in a total of 25 ounces. Since there were originally 2 ounces of salt in the solution (10% of 20) and only water was added, there are still 2 ounces of salt. But 2 ounces out of 25 ounces is $\frac{2}{25}$, which is equal to $\frac{8}{100}$ or 8%. Then this choice is correct.

It is not actually necessary to do this arithmetic to answer the question that you were asked. When water is added to a solution, it dilutes the solution; that is, it lowers the percent of dissolved substance. In this case, the percent salt of the final mixture would have to be less than 10%. Please proceed to question 3 below.

56
2

Question 3

An alloy of 50 pounds of iron and tungsten is 2% tungsten. If 5 pounds of tungsten are added to the alloy, perform the calculations to find the number of pounds of tungsten in the final mixture.

(A) 55 $\quad$ (B) 7

(C) 6 $\quad$ (D) 5

If you calculate 15% of 20 , you will get 3 .
You can then proceed to do more arithmetic with this number, but what has that to do with this problem?

We are interested in the amount of solution, rather than in the amount of pure water or pure salt.

Please return to page 51/2 and try question 7 again.

If we organize our work in the usual form, we have:

I THE VARIABLES:

	Number of Pounds of alloy	· Percent Copper	= Number of pounds of Copper
1.	x	100	1(x)
2.	480 - x	4	.04(480 - x)
Mix.	480	10	.10(480)

Note that (480 - x) represents the amount of the 4% alloy.

II THE RELATIONSHIP:

The total number of pounds of copper in the mix is equal to the sum of the two amounts in the two alloys. This can be interpreted in the diagram. In the third column, the sum of the first two lines equals the third. This fact gives us the equation we need, and solving it, we find that this choice is not correct.

Please return to page 70/2 and try question 15 again.

58/1

If there are 70 cc. of one solution, there are 30 cc. of the other.

Then we would have:

	Number of cc. of Solution	· Percent Salt	= Number of cc. of Salt
1.	70	5	3.5
2.	30	10	3
M.	100	7	7

The total number of cc. in the two solutions should equal the amount in the mixture. This is located in the last column.

But,

$$3.5 + 3$$

does not equal 7 . Then this choice is not correct.

Please return to page 52/2 and try question 11 again.

58/2

Since this equation contains the variable in the denominator of at least one fraction, it <u>is</u> a fractional equation.

Therefore, this choice is not correct.

Return to page 83/1 and try question 1 again.

The original solution contained 20 cc. of acid out of a total of 100 cc. If there were 25 cc. of solution left, we would have 20 cc. out of a total of 25 cc. as a result. But this is not 25%. Therefore, this choice is not correct.

Please return to page 55/2 and try question 13 again.

In order to get 120 gallons, using n gallons of cream, he will need (120 - n) gallons of milk. Then:

	Number of Gallons of Solution	• Percent Butterfat	= Number of Gallons of Butterfat
1.	n	20	.20(n)
2.	(120 - n)	2	.02(120 - n)
M.	120	35	.035(120)

Solving this equation will show you that this choice is not correct. The equation can be constructed from the data in the last column.

Please return to page 77/1 and try question 16 again.

$\frac{60}{1}$

The problem stated that the original solution had acid in it.

Since only water was removed, the final solution still has acid in it.

Then this choice is not correct.

Please return to page $\frac{54}{2}$ and try question 9 again.

$\frac{60}{2}$

The (L C D) of an equation must meet two requirements:

(1) Each denominator must divide into it; that is, it is a multiple of each denominator.

(2) It must be the lowest possible such number.

Is the denominator $5x$ divisible by 4 ? Since it is not, this cannot be a common denominator, and it is certainly not the lowest common denominator.

Please return to page $\frac{68}{2}$ and try question 2 again.

I THE DATA

It is simplest to take x as the number of cc. of water evaporated. Then we have:

	Number of cc. of Solution	· Percent Acid	= Number of cc. of Acid	III: CHECK:		
1.	100	20	.20(100)	100	.20	20
2.	x	0	0 evaporate	20	0	0
M.	(100 - x)	25	.25(100 - x)	80	.25	20

Note: Evaporation removes water, which gives the minus sign.

II THE RELATIONSHIP:

The amount of acid in the original solution was not changed when the water evaporated. Then:

$$.25(100 - x) = .20(100) \quad [\text{D}$$

$$25 - .25x = 20 \quad \swarrow - 25$$

$$-.25x = -5 \quad \swarrow (-100)$$

$$25x = 500 \quad \swarrow \div 25$$

$$x = 20$$

Since 20 cc. of water was removed from 100 cc., the result is 80 cc.; which is, therefore, the correct choice.

Please proceed to question 14 below.

Question 14

How much salt must be added to 40 pounds of a 5% solution to produce a 24% solution? Apply your knowledge to find the set containing the correct answer.

(A) $\{30 , 35 , 40\}$ (B) $\{15 , 20 , 25\}$

(C) $\{18 , 26 , 34\}$ (D) $\{4 , 10 , 16\}$

62
1

Calculating 2% of 50 we find that there was 1 pound of tungsten in the original alloy. Then adding the 5 pounds gives us the correct total of 6 pounds of tungsten at the end. It is interesting to note that there are now 55 pounds of alloy, so that the tungsten is $\frac{6}{55}$ or approximately 11% of the total.

To organize your thoughts make use of the chart to relate the given data.

	N	x R	= I
Alloy	50	.02 .	.02(50)
Added	5	1.00	5
Mixture	55	=	5 + (.02)(50)
		=	5 + 1
		=	6

All you were asked for was the amount of tungsten in the new mixture.

Please proceed to question 4 below.

62
2

Question 4

Two quarts of milk containing 2% butterfat is mixed with 20 quarts of milk containing 5% butterfat. Perform the calculations to find the amount of milk in the mixture.

(A) 22

(B) 20

(C) 2

(D) None of these.

There were 20 quarts of solution when we started. Since 2 quarts were evaporated (removed), the total amount of solution was decreased leaving 18 quarts remaining.

There are many other questions that could be asked about this situation, but you answered this one correctly.

Please proceed to question 8 below.

Question 8

If 3 pints of water are evaporated from 15 pints of a 20% solution of acid, what is the percent acid in the remaining solution? Apply your knowledge to find the correct choice.

(A) 0%

(B) 20%

(C) 25%

(D) 30%

$\frac{64}{1}$

If there are 40 cc. of the 5% solution, there will be 60 cc. of the 10% solution. Then we have:

	Number of cc. of solution	• Percent Salt	= Number of cc. of Salt
1.	40	5	2
2.	60	10	6
MIXTURE	100	7	7

The number of cc. of salt in the mixture is made up of the number of cc. of salt in the two given solutions (last column).

But 2 + 6 does not equal 7 .

Then this choice is not correct.

Please return to page $\frac{52}{2}$ and try question 11 again.

$\frac{64}{2}$

The first step in finding the LCD is factoring the denominators, which gives us:

$$\frac{3x}{2(x-3)} + \frac{1}{2} = \frac{5}{(2)(3)(x)}$$

The factor 2 appears in each denominator, but it should be used only once in calculating the LCD since its highest exponent is one.

Please return to page $\frac{85}{2}$ and try question 3 again.

$\frac{65}{1}$

The original solution contained 20 cc. of acid out of a total of 100 cc. If there were 20 cc. of solution left, we would have 100% acid, not 25% . Then this choice is not correct.

Please return to page $\frac{55}{2}$ and try question 13 again.

$\frac{65}{2}$

The Multiplication Property of Equality states that if

$$a = b \text{ then } ac = bc$$

Did you forget to multiply the right side of the equation? Either you forgot, or you made an error in the multiplication since this choice is not correct.

Please return to page $\frac{99}{2}$ and try question 5 again.

66
1

The original solution contained 20 pints of which 15% were acid. Multiplying 15% by 20 , tells us that there were 3 pints of acid in the original solution. When water is evaporated, it is pure water; there is no acid removed. Therefore, this choice is correct.

Please proceed to question 10 below.

66
2

Question 10

Apply your knowledge to find how many pounds of a 20% salt solution should be mixed with 30 pounds of a 5% solution to produce a mixture which contains 10% salt. If n is the number of pounds, which is the correct statement?

(A) $n < 6$

(B) $6 < n < 12$

(C) $12 < n < 18$

(D) $18 < n < 24$

Adding the two solutions together gives us a total of 50 pounds. Since it is 6% salt, we take 6% of 50 and find that there are 3 pounds of salt in the final solution. Then this choice is correct.

	N	x	R	=	I
1.	20		.05		.05(20)
2.	30				
MIX.	50	x	.06	=	.06(50)

Note: We could not find how much salt was disolved in the second solution directly. But now having found the total amount of salt in the final mixture, we can subtract the amount contained in the first solution and then determine the amount in the second and also determine the percent salt in the second solution.

Please proceed to question 6 below.

Question 6

A 20% sugar solution is mixed with a 10% sugar solution, and the result is 50 gallons of a 32% sugar solution. How much of the 20% solution was used? Choose the correct statement about the answer.

(A) 25 gallons of 20% solution were used.

(B) More than 25 gallons of 20% solution were used.

(C) Not enough information is given to determine an answer.

(D) The question has no answer; the problem is impossible.

$\frac{68}{1}$

The equation does contain a fraction. However the denominator does not contain the variable. Then this is not a fractional equation. If you insist on giving it a name, you may call it an equation with fractional coefficients. Your text defines a fractional equation as one which contains the variable in a denominator.

Since you were asked to find the equation which is not fractional, this choice is correct.

Please proceed to question 2 below.

$\frac{68}{2}$

Question 2

Perform the calculation to find the LCD for the equation:

$$\frac{2}{x} - \frac{3}{5x} = \frac{1}{4}$$

(A) $5x$

(B) $5x^2$

(C) $20x^2$

(D) $20x$

If there are 60 cc. of the 5% solution, there will be 40 cc. of the 10% solution. Then we have:

	Number of cc. of Solution	• Percent Salt	= Number of cc. of Salt
1.	60	5	3
2.	40	10	4
M.	100	7	7

The number of cc. of salt in the mixture is found by adding the amounts of salt in the two solutions given. This is verified in the last column. Since

$$3 + 4 = 7$$

The same number must agree with the product of the amount and percent salt of the mixture. This is verified in the last row.

$$100 \times .07 = 7$$

Please proceed to question 12 below.

Question 12

A mouthwash is supposed to contain 2% ingredient X, but the manufacturer finds that he has 1000 gallons containing 5% ingredient X. How much water should he add to dilute the mixture to the desired strength? Apply your knowledge to find the correct statement about "n", the number of gallons of water to be added.

(A) $n < 200$ (B) $200 < n < 700$

(C) $700 < n < 1200$ (D) $n > 1200$

$\frac{70}{1}$

I THE VARIABLES

Letting x be the number of pounds of salt to be used, we have:

	Number of Pounds of Solution	• Percent Salt	= Number of Pounds of Salt
1.	40	5	.05(40)
2.	x	100	1(x)
M.	(40 + x)	24	.24(40 + x)

II THE RELATIONSHIP:

The number of pounds of salt in the first solution plus the number of pounds added is the same as the amount in the final mixture.

$$
\begin{aligned}
.24(40 + x) &= .05(40) + 1(x) && [\ D \\
9.6 + .24x &= 2. + x && \times(100) \\
960 + 24x &= 200 + 100x && - 24x \\
960 &= 200 + 76x && - 200 \\
760 &= 76x && \div\ 76 \\
10 &= x
\end{aligned}
$$

III CHECK

N	R	I
40	.05	2
10	1	10
50	.24	12

Then this choice is correct.

Please proceed to question 15 below.

$\frac{70}{2}$

Question 15

In order to produce 480 pounds of an alloy that is 10% copper, a manufacturer is going to add pure copper to an alloy containing 4% copper. Apply your knowledge to find the correct statement about x, the number of pounds of pure copper to be added.

(A) $x < 25$

(B) $25 < x < 50$

(C) $50 < x < 75$

(D) $x > 75$

You have overlooked an important step; namely, factoring all the denominators. While many students can handle simple problems without writing down the factors, you made a mistake here. Surely you realize that $(2x - 6)$ is not divisible by $6x$.

Please return to page 85/2 and try question 3 again.

This choice indicates that you found the proper (L C D), but your derived equation is not correct.

Your error is due to a careless application of the DISTRIBUTIVE PROPERTY.

Please return to page 97/2 and try question 7 again.

Let's review the procedure.

I THE VARIABLES:

Using n gallons of cream, he will need (120 - n) gallons of milk. Then:

	Number of Gallons of Solution	Percent Butterfat	= Number of Gallons of Butterfat
1.	n	20	.20(n)
2.	(120 - n)	2	.02(120 - n)
M.	120	3.5	.035(120)

II THE RELATIONSHIP:

The total number of gallons of butterfat is composed of the number of gallons in each of the two components. The last column gives all the necessary data.

.20n + .02(120 - n)	=	.035(120)	[D
.20n + 2.4 - .02n	=	4.2	[·(100)
20n + 240 - 2n	=	420	[C
18n + 240	=	420	✓-240
18n	=	180	✓÷18
n	=	10	

III CHECK:

N	R	I
10	.20	2.0
110	.02	2.2
120	.035	4.2

Then this choice is correct.

Now go on to page 73
1

You have completed this Segment. Hand in the PUNCH CARD. You should have entered in your NOTEBOOK the form which we have been using in these Percent Mixture problems.

	Number of Units of Solution	·	Percent Ingredient	=	Number of Units of Ingredient
First					
Second					
Total					

This can be reduced to:

	N	x	R	=	I
1.					
2.					
M.					

You should now be able to complete ASSIGNMENT 12 problems 5-8 .

.....

The LCD must be the lowest multiple of all of the denominators.

There are two reasons why this choice is not correct.
First, this choice is not divisible by the denominator 4 and it, therefore, cannot be a common denominator.
Can you find the second reason?

Please return to page 68/2 and try question 2 again.

$\frac{74}{1}$

Almost, but not quite correct.

What happened in your multiplication on the right side of the equation?

Note: $12x^2 \cdot \frac{1}{4x^2} = \frac{3 \cdot 4 \cdot x^2}{1 \cdot 4 \cdot x^2}$

$= 3$

Please return to page $\frac{99}{2}$ and try question 5 again.

$\frac{74}{2}$

In solving equation P, the correct procedure is to multiply by the common denominator, $2(y + 1)$, which results in equation Q.

If we now solve equation Q, we will have all the solutions of equation P and perhaps some extra solutions.

Solving equation Q yields a single solution. All that you need do is to check it in equation P and in equation Q. When you do this, you will find that this choice is not correct.

Please return to page $\frac{88}{2}$ and try question 8 again.

Since neither denominator can be factored, the (L C D) is

$$3(m + 3)$$

Did you use $(m + 3)$ as the L C D ? Since it is not divisible by 3 , it is not a common denominator.

When the fraction $\frac{1}{3}$ is multiplied by $(m + 3)$, the denominator will not reduce to ONE.

Please return to page $\frac{87}{2}$ and try question 6 again.

Since the equation on this line may not be equivalent to the original equation; that is, a new root may have been introduced through the multiplication, it does not prove the acceptability of the answer for the original problem. It merely checks the answer for that line and all that follows it.

Please return to page $\frac{98}{2}$ and try question 10 again.

Let us review the procedure that we think you should follow.

I THE VARIABLES:

If x is the amount of pure copper, (480 - x) is the amount of alloy since the total number of pounds of the mixture is 600. Then we have:

	Number of Pounds of Alloy	• Percent Copper	= Number of Pounds of Copper	III CHECK: N • R I
1.	x	100	1(x)	30 1 30
2.	(480 - x)	4	.04(480 - x)	450 .04 18
MIX.	480	.10	.10(480)	480 .10 48

II THE RELATIONSHIP:

The total amount of copper in the mixture is the same as the amount in the original alloy plus the amount of pure copper added. The third column lists the necessary data. From this we can write the equation,

$$1(x) + .04(480 - x) = .10(480) \quad [\ D$$
$$x + 19.2 - .04x = 48 \quad \swarrow (100)$$
$$100x + 1920 - 4x = 4800 \quad [\ C$$
$$1920 + 96x = 4800 \quad \swarrow -1920$$
$$96x = 2880 \quad \swarrow \div 96$$
$$x = 30$$

Then this choice is correct, since it falls within the range

$$25 < x < 50$$

Please proceed to question 16.

Question 16

A farmer has some milk which is 2% butterfat. How much cream, which is 20% butterfat, should he add to produce 120 gallons of milk containing 3.5% butterfat? Apply your knowledge to find the correct statement about n , the number of gallons of cream to be added.

(A) $n < 10$

(B) $10 < n < 50$

(C) $50 < n < 90$

(D) $n > 90$

True, the expression you chose is a common denominator, but not the lowest common multiple of the denominators.

The (L C D) is found as the product of factors, but all you have done is multiply the complete denominators. In this problem, everyone needs to factor first.

Please return to page 90/2 and try question 4 again.

78/1

This is a satisfactory common denominator since it is divisible by the first denominator x and also by the second and third denominators, $5x$ and 4. However, this is not the correct choice since it is not the lowest common denominator.

Please return to page 68/2 and try question 2 again.

78/2

Whenever both sides of an equation are multiplied by the same quantity, whatever it may be, the result is an equation, although it may not be one that is helpful in finding the solution set.
In any event, this choice is not correct.

Please return to page 97/2 and try question 7 again.

You appear to be consistent in making the same mistake in each multiplication. What happened to the denominators of the fractions; did you just ignore them?

Please return to page 99/2 and try question 5 again.

If you solve equation S , which is a quadratic equation, you will find two values for x .

$x^2 - x - 2 = 0$ Factor

$(\quad)(\quad) = 0$ Let each factor equal 0 .

It is then necessary to check these values in equation R . When you do this correctly, you find that this choice is not correct. The "null set", $\emptyset$, contains no elements.

Please return to page 94/2 and try question 9 again.

80/1

The multiplication Property of Equality has been misapplied.
You have forgotten to multiply both sides of the equation by the LCD.
This must be done regardless of whether the term is a fraction or not.

Please return to page 87/2 and try question 6 again.

.....

80/2

The fact that a solution to a fractional equation may not check in the original equation does not depend on just being careful.
All the steps might really be mathematically correct.
Nevertheless, when you multiply both sides of an equation by a quantity containing the variable, your new equation may have a solution that did not exist on the line above. The only positive check is to substitute the values you find when you complete the solution in the original problem.

Then this choice is not correct.

Please return to page 98/2 and try question 10 again.

You have started correctly, factoring the denominators and then setting up the product of factors for the LCD. However, it appears that you have forgotten an important item. Each factor is used only once unless it repeats in a single denominator, that is, if it has an exponent other than one.
Each different factor must be used with the highest exponent that appears with it.

Please return to page 90/2 and try question 4 again.

.....

In order to solve this equation, it is necessary to find the LCD. None of the denominators can be factored; therefore, the LCD is

$$3x(x + 3)$$

Multiplying by this should give the equation

$$5(3)(x + 3) - 1(x)(x + 3) = 8(3)(x)$$

If you now proceed to solve this equation, which is quadratic, you will find two values of x. However, you must check them in the original equation before you know if one or both of them are solutions. You should find that this choice is not correct.

Please return to page 91/2 and try question 12 again.

Volume 12 Segment 3 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	1 3	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	1 2	(Volume Number)
	66	and	68	0 3	(Segment Number)

Preliminary Notes

In order to solve a fractional equation, that is, an equation containing the variable in a denominator, we begin by removing the fractions. You will recall that we can multiply both sides of an equation by the same quantity, and the results will still be equal. Of course, we do not use just any multiplier; it must be one which contains each denominator as a factor, that is, it must be a common multiple of the denominators. We call the smallest of the common multiples the

Lowest Common Denominator, or LCD.

For the equation

$$\frac{60}{z^2 - 36} + 1 = \frac{5}{z - 6}$$

we first factor the denominators (where possible) getting:

$$\frac{60}{(z + 6)(z - 6)} + 1 = \frac{5}{z - 6}$$

Now we are in a position to build the common denominator as a product of the different factors which we see. Thus the LCD is

$$(z + 6)(z - 6)$$

and we can now multiply both sides of the equation by this quantity.

You should now review your READING ASSIGNMENT for this Segment, Pages 312-313 .

Now go on to page 83/1 .

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Which of the choices do you recognize as NOT a fractional equation?

(A) $\frac{5}{x} + \frac{2}{3x} = 7$

(B) $\frac{y}{3} = \frac{9}{y}$

(C) $\frac{1}{2}x + 2 = 5$

(D) $\frac{x}{x - 5} + 1 = \frac{x}{3}$

.....

This choice is not correct. Did you multiply both sides of the equation by the LCD, which is

$(x + 2)(x - 5)$?

One of the other choices is correct.

Please return to page $\frac{97}{2}$ and try question 7 again.

84/1

In the procedure for solving a fractional equation, you obtain an equation which contains no fractions. This derived equation will always be satisfied by all the roots of the original equation, and perhaps by some additional roots.

To be a proper subset, the solution set for S would have to contain only some of the members of the solution set for equation R .

Therefore, this choice is not correct.

Please return to page 94/2 and try question 9 again.

.....

84/2

If you multiply both sides of this equation by the LCD, which is

$$(x + 5)(x - 5)$$

you will get an equation which can be solved. After solving the equation, remember that you must check any solution in the original equation. When you do this, you will find that this choice is not correct.

Please return to page 101/2 and try question 13 again.

This is a satisfactory common denominator since it is a multiple of each of the denominators and is, therefore, divisible by each of them.

Let's review a "working method" for determining the Least Common Multiple (LCM) of the denominators. (The LCM is also called the Least Common Denominator, or LCD when it refers to the denominators of fractions.)

(1) Factor each denominator and place the factors in base-exponent form.
(2) Form the LCD by multiplying each of the different bases using the highest exponent that appears for each base.

In this case, the factors of the different denominators are x, $5x$, , and 2^2, respectively.

We form the LCD according to the rule and get

$$2^2 5x \quad \text{or} \quad 20x$$

Now proceed to question 3 below.

.....

Question 3

Apply your knowledge to find the LCD for the equation:

$$\frac{3x}{2x - 6} + \frac{1}{2} = \frac{5}{6x}$$

(A) $12x(2x - 6)$
(B) $6x(2x - 6)$
(C) $6x(x - 3)$
(D) $2x - 6$

86
1

It is only necessary to check the values offered in this choice to find out if they are correct. There is very little arithmetic to be done before you discover that this choice is not correct. Of course, you must remember that any value which makes a denominator equal to zero is not acceptable.

Please return to page 105 and try question 11 again.
2

.....

86
2

It would appear that you have done an excellent job.
Your only error seems to be that you have answered the wrong question.
You were asked to find the value on the right side of the equation, when you have substituted the correct value for the variable y .

Please return to page 93 and try question 14 again.
2

When the Multiplication Property of Equality is used, you must be alert to the possibility of having to use the Distributive Property and to use the methods for reducing fractions.

Let's review the steps you should have taken.

$$\frac{1}{3x} - \frac{2}{3} = \frac{1}{4x^2} \qquad \text{[illegible } 12x^2$$

$$12x^2\left(\frac{1}{3x} - \frac{2}{3}\right) = \frac{1}{4x^2} \cdot 12x^2 \qquad \text{[D}$$

$$\frac{12x^2}{3x} - \frac{24x^2}{3} = \frac{12x^2}{4x^2} \qquad \text{[Reduce}$$

$$4x - 8x^2 = 3$$

Then this choice is correct.

Please proceed to question 6 below.

.....

Question 6

In solving the equation

$$\frac{2m}{m+3} + \frac{1}{3} = 2$$

we multiply both sides by the LCD and get one of the following equations. Apply your knowledge to find which is the correct derived equation.

(A) $2m + m = 2m + 6$

(B) $6m + m + 3 = 6m + 18$

(C) $6m + m + 3 = 2$

(D) None of these.

$\frac{88}{1}$

Very good. This choice is correct.

$$\frac{1}{x+2} - \frac{1}{x-5} = \frac{1}{x^2 - 3x - 10}$$

[Since the 3rd denominator can be factored into $(x + 2)(x - 5)$ and it is composed of the other two denominators, it is the LCD

$$\frac{(x+2)(x-5)}{(x+2)} - \frac{(x+2)(x-5)}{(x-5)} = \frac{(x+2)(x-5)}{(x+2)(x-5)}$$

$$(x-5) - (x+2) = 1 \quad \text{[D}$$

$$x - 5 - x - 2 = 1$$

Please proceed to question 8 which follows.

.....

$\frac{88}{2}$

Question 8

Given equation (P) $\frac{5}{y+1} = \frac{3}{2y+2} + 1$

and equation (Q) $10 = 3 + 2y + 2$

We derive equation Q by multiplying equation P by $(2y + 2)$.

Apply your knowledge to find which choice is correct.

(A) P and Q have the same solution set.

(B) The solution set for P is a proper subset of the solution set for Q .

(C) The solution set for Q is a proper subset of the solution set for P .

(D) The sets are not the same but they have the same number of elements; that is, they are equivalent.

2

When we solve equation S, we get two values for x.
On checking each value in equation R and in equation S, we find that the two values do not check in both equations. Then this choice is not correct.
The sets are not the same.

Please return to page $\frac{94}{2}$ and try question 9 again.

.....

We multiply by the LCD, which equals

$$(x - 2)(x + 1)$$

and the resulting equation is:

$$2x(x + 1) + 3(x - 2) = 2x^2 - x$$

When we solve this equation, and check the solutions in the original equation, we find that this choice is not correct.

Please return to page $\frac{110}{2}$ and try question 15 again.

$\frac{90}{1}$

The first step in finding the LCD is factoring the denominators, which gives us:

$$\frac{3x}{2(x - 3)} + \frac{1}{2} = \frac{5}{(2)(3)(x)}$$

Since we do not use a factor more than once, even though it appears in another denominator, if its highest exponent is 1, only <u>one 2</u> will be used. Then the LCD is

$$(2)(x - 3)(3)(x)$$

which can be rewritten as

$$6x(x - 3)$$

Therefore, this choice is correct.

Please proceed to question 4 below.

.....

$\frac{90}{2}$

<u>Question 4</u>

Apply your knowledge to find the LCD for the equation:

$$\frac{x + 2}{2x + 6} + \frac{3}{x - 1} = \frac{5x^2 + 7}{x^2 + 2x - 3}$$

(A) $(2x + 6)(x - 1)(x^2 + 2x - 3)$

(B) $2(x + 3)(x - 1)(x + 3)(x - 1)$

(C) $(2x + 6)(x - 1)(x + 3)$

(D) $2(x - 1)(x + 3)$

The only values offered in the other choices are 0 , 1 , and -1 .

If x is 1 or -1 , x^2 equals 1 , and $x^2 - 1$ equals 0 .

Since a denominator cannot have the value zero, this rules out the values 1 or -1 for x .

We also find by substituting zero for x in the original equation, that zero is not a solution.

Then this choice is correct.

Please proceed to question 12 below.

Question 12

Apply your knowledge to solve the equation

$$\frac{5}{x} - \frac{1}{3} = \frac{8}{x + 3}$$

Which is the correct statement about the value of x which satisfies the equation?

(A) There is only one solution, and it is positive.

(B) There is only one solution, and it is negative.

(C) There are two solutions which have different signs.

(D) There are two solutions which have the same sign.

$\frac{92}{1}$

You have missed the correct choice.
What did you use as the LCD? Since the first denominator cannot be factored, the LCD should be 3(m + 3). When the denominators are not factorable, then the LCD is merely the product of the denominators.

Please return to page $\frac{87}{2}$ and try question 6 again.

.....

$\frac{92}{2}$

If we multiply both sides by 5m, which is the LCD, we get a first degree equation. Solving this equation, we find a value which checks in the original equation. However, it does not agree with this choice.

Please return to page $\frac{109}{1}$ and try question 17 again.

Yes, it might seem strange, but there is no solution.

It is an impossible problem.

Factoring the last denominator, we find that the LCD is $(x + 5)(x - 5)$

$$\frac{1}{x - 5} + \frac{1}{x + 5} = \frac{10}{x^2 - 25} \quad \times (x + 5)(x - 5)$$

$$x + 5 + x - 5 = 10 \quad [\text{ Collect}$$

$$2x = 10 \quad \div 2$$

$$x = 5$$

If we substitute $x = 5$ in the original equation, we get two denominators equal to zero. But this indicates that the value we used is not a solution. Then there is no solution, and this choice is correct.

Please proceed to question 14 below.

Question 14

Solve the equation

$$\frac{2y}{y + 3} - \frac{5}{2} = \frac{1}{y + 3}$$

If the correct value of y is substituted on the right side of the equation, apply your knowledge to find the value of the right side.

(A) -17

(B) 2

(C) $\frac{1}{16}$

(D) $-\frac{1}{14}$

94
1

It is important to remember that the procedure we use for solving fractional equations gives us an equation without fractions as our second step. This second equation always has all the roots of the original fractional equation, but it may have some new ones. Therefore, it is important to check the solution set of the derived equation in the original equation. This will allow you to determine whether some of the values of the solution set should be rejected as "extraneous" roots.

We solve equation Q , getting $y = 2.5$. Now if we substitute this in equation P , we find that it checks (giving $\frac{10}{7} = \frac{10}{7}$). We substitute in equation Q , and again we find that it checks. Therefore, the solution set for each equation is 2.5 , and this choice is correct.

Please proceed to question 9 below.

.....

94
2

Question 9

In solving the equation

$$(R) \quad \frac{1}{x + 1} = 1 - \frac{2}{x^2 - 1}$$

We obtain the equation

$$(S) \quad x - 1 = x^2 - 1 - 2$$

by multiplying both sides by the LCD . Apply your knowledge to find which statement is true.

(A) The solution set for R is $\emptyset$.

(B) The solution set for S is a proper subset of the solution set for R .

(C) The solution set for R is a proper subset of the solution set for S .

(D) The solution sets are the same.

This is almost the correct result.

If you factored each denominator and then set up the product of all the factors being careful not to use a factor too many times, you would have had the correct choice.

Please return to page 90/2 and try question 4 again.

Since the LCD is 2y we multiply by it and get an equation with no fractions. But this equation is a first degree equation and has only one solution.

Therefore, this choice cannot be correct.

Please return to page 102/2 and try question 18 again.

96
1

In this problem the LCD is $(x - 2)(x + 1)$ and we multiply by it, getting the equation:

$$\begin{aligned} 2x(x + 1) + 3(x - 2) &= 2x^2 - x \\ 2x^2 + 2x + 3x - 6 &= 2x^2 - x \\ 2x + 3x - 6 &= -x \\ 5x - 6 &= -x \\ 6x &= 6 \\ x &= 1 \end{aligned}$$

[D
$-2x^2$
[C
$+x, +6$
$\div 6$

Now, we must check this solution in the original equation, which gives:

$$\frac{2}{-1} + \frac{3}{2} = \frac{1}{-2}$$

But $-2 + \frac{3}{2}$ does equal $-\frac{1}{2}$. Then this choice is correct.

Please proceed to question 16 below.

.....

96
2

Question 16

Apply your knowledge to solve the equation

$$\frac{2x}{x^2 - 9} + \frac{1}{x^2 - 4x + 3} = \frac{2}{x - 1}$$

Which statement about the solution is correct?

(A) There is no solution.
(B) There is a solution which is a positive integer.
(C) There is a solution which is a negative integer.
(D) There is a solution which is a fraction.

$\frac{97}{1}$

Good. Let's review the problem step by step.

$$\frac{2m}{m+3} + \frac{1}{3} = 2 \qquad \text{« } 3(m+3), \text{ the LCD}$$

$$\frac{3(m+3)\cdot 2m}{(m+3)} + \frac{3(m+3)}{3} = 3(m+3)\cdot 2 \qquad \text{[Reduce, + D}$$

$$6m + (m+3) = 6m + 18 \qquad \text{[Combine}$$

$$7m + 3 = 6m + 18 \qquad \text{« } -6m, -3$$

$$m = 15$$

Therefore, this choice is correct.

Please proceed to question 7 below.

$\frac{97}{2}$

Question 7

If you apply the principle of multiplying both sides of the equation

$$\frac{1}{x+2} - \frac{1}{x-5} = \frac{1}{x^2-3x-10}$$

by the LCD, which is the resulting derived equation?

(A) $x - 5 - x + 2 = 1$

(B) $x - 5 - x - 2 = 1$

(C) There is none.

(D) There is one, but it is not listed.

98
1

When we solve equation S , we get the values $x = 2$, and $x = -1$ Substituting $x = 2$ in equations R and S , we find that it checks in both. However $x = -1$ does not check in equation R , since we get meaningless fractions with zero denominators. But it does check in equation S . Then the solution set of R is $\{2\}$, while the solution set for S is $\{-1, 2\}$.

Therefore, the solution set for R is a proper subset of the solution set for S .
Then this choice is correct.

Please proceed to question 10 below.

.....

98
2

Question 10

The solution of the following fractional equation can be done as follows:

(1)	$10 - \frac{x}{x+5} = \frac{5}{x+5}$	$[\cdot (x+5)$
(2)	$10(x+5) - x = 5$	[D
(3)	$10x + 50 - x = 5$	[Collect
(4)	$50 + 9x = 5$	$\propto -50$
(5)	$9x = -45$	$\propto \div 9$
(6)	$x = -5$	

Choose the statement which describes the correct method of checking the solution.

(A) Substitute the value for x on line 1 .

(B) Substitute the value for x on line 2 .

(C) Substitute the value for x on any line.

(D) The proper method is to repeat the steps in the solution very carefully.

The first step is to factor denominators, which would result in the equation

$$\frac{x + 2}{2(x + 3)} + \frac{3}{(x - 1)} = \frac{5x^2 + 7}{(x + 3)(x - 1)}$$

We note that there are two factors which appear in more than one denominator, but we take them only once in setting up the LCD, since each has 1 for an exponent.

Then the LCD is

$$2(x + 3)(x - 1)$$

and this choice is correct.

Please proceed to question 5 below.

.....

Question 5

When both sides of the equation

$$\frac{1}{3x} - \frac{2}{3} = \frac{1}{4x^2}$$

are multiplied by $12x^2$, we obtain an equation with no fractions. Apply your knowledge to find which choice is the resulting equation.

(A) $4x - 8x^2 = 3$

(B) $4x - 8x^2 = 1$

(C) $4x - 8x^2 = 3x^2$

(D) $12x^2 - 24x^2 = 1$

100/1

When you were asked to find the value of the right side of the equation, which line of your work did you use? Remember that the value of one side of an equation changes as you proceed from one equation to the next equivalent equation. The substitution you were asked to do should be done in the original equation.

Please return to page 93/2 and try question 14 again.

.....

100/2

After factoring the denominators, we find that the LCD is

$$(t - 2)(2t + 1)$$

After multiplying by the LCD, and simplifying, we get a quadratic equation:

$$2t^2 - t - 1 = 0$$

Solving the resulting quadratic equation, we have two values of t which must be checked. The check shows that this choice is not correct.

Please return to page 113/2 and try question 19 again.

Since the denominators cannot be factored, the LCD is

$$3x(x + 3)$$

Multiplying by this LCD, we get the equation

$$5(3)(x + 3) - 1(x)(x + 3) = 8(3)(x) \qquad [\ \text{D}$$

$$15x + 45 - x^2 - 3x = 24x \qquad [\ \text{Collect}$$

$$+ 45 - x^2 + 12x = 24x \qquad \propto -24x$$

$$- x^2 - 12x + 45 = 0 \qquad \propto \cdot (-1)$$

$$x^2 + 12x - 45 = 0$$

Factoring and solving, we find that $x = -15$, and $x = 3$.
Substituting $x = -15$ in the original equation, we get

$$-\frac{1}{3} - \frac{1}{3} = \frac{2}{-12}$$

which checks.

Then this choice is correct.

Please proceed to question 13 below.

Question 13

Apply your knowledge to solve the equation

$$\frac{1}{x - 5} + \frac{1}{x + 5} = \frac{10}{x^2 - 25}$$

Which statement about the solution is correct?

(A) $x \leq -5$

(B) $-5 < x < 5$

(C) $x \geq 5$

(D) There is no such solution.

102
1

If we multiply both sides of the equation by 5m which is the LCD , we get:

$$10 + m = 5m \qquad \propto - m$$

$$10 = 4m \qquad \propto \div 4$$

$$\frac{10}{4} = m \qquad \text{[Reduce, and SYMMETRY PROPERTY}$$

$$m = \frac{5}{2}$$

Checking, we have:

$$\frac{2}{\frac{5}{2}} + \frac{1}{5} = 1 \qquad \left[\begin{array}{l} 2 \div \frac{5}{2} = \\ 2 \cdot \frac{2}{5} = \frac{4}{5} \end{array}\right.$$

so that we have

$$\frac{4}{5} + \frac{1}{5} = 1$$

which checks.

Then this choice is correct.

Please proceed to question 18 below.

.....

102
2

Question 18

Apply your knowledge to solve the equation

$$\frac{3}{y} - \frac{5}{2y} = 1$$

Which statement about the solution is correct?

(A) There are two solutions.

(B) There is only one solution, and it is an integer.

(C) There is only one solution, and it is a positive fraction.

(D) There is only one solution, and it is a negative fraction.

Factoring denominators, we get:

$$\frac{2x}{(x - 3)(x + 3)} + \frac{1}{(x - 3)(x - 1)} = \frac{2}{x - 1}$$

Then we multiply by the LCD,

$$(x - 3)(x + 3)(x - 1)$$

and get an equation which we solve. Checking the solution in the original equation tells us that this choice is not correct.

Please return to page $\frac{96}{2}$ and try question 16 again.

Before doing any arithmetic with numbers, it is wise to examine their units. Did you notice that the number 2 was in hours, while the number 30 was in minutes? You cannot do any meaningful arithmetic until you express them in the same units.

Please return to page $\frac{135}{2}$ and try question 3 again.

104/1

You should consider the choice more carefully.

If he graded one-fifth of a paper in one hour, then he would grade one whole paper in five hours. That is not what the problem stated.

Please return to page 123/1 and try question 1 again.

104/2

What did you get for the rate of work of the pump?

If the pump took 2 hours to empty the tank, its rate of work would be $\frac{1}{2}$ of a tank per hour. Once you get the correct rate of work, you should be able to complete the problem correctly.

Please return to page 120/2 and try question 4 again.

Very good. This choice is correct.

It is very important to remember that line two may not have an equation equivalent to the original on line 1 . That is why we must check on line 1 .

It happens that we get zero denominators, which means that this value does not check; that is, -5 is not a solution of the original equation.

No matter how carefully you work, you cannot prevent an extraneous value from appearing in your result. The only way you can detect it is by checking.

Please proceed to question 11 below.

.....

Question 11

Perform the calculations necessary to find the set containing all the solutions of the equation

$$\frac{1}{x+1} + \frac{1}{x-1} = \frac{2}{x^2-1}$$

(A) $\{1, -1\}$

(B) $\{0, -1\}$

(C) $\{0, 1\}$

(D) None of these.

106
1

In solving this equation, it is necessary to multiply by the LCD which is

$$2(y + 3)$$

The resulting equation is:

$$2y(2) - 5(y + 3) = 1(2)$$

If you continue carefully from this point, you should be able to avoid the error which you made. This choice is not correct

Please return to page 93 and try question 14 again.
2

.....

106
2

This problem has a complication in that the factors of the first denominator are

$$2(x - 5)$$

while the second denominator equals

$$(5 + x)(5 - x)$$

We can avoid the difficulty of

$$x - 5$$

in one place and

$$5 - x$$

in the other by factoring out (-1) in the second denominator. Thus, we have:

$$\frac{x - 3}{2x - 10} + \frac{10}{25 - x^2} = 1$$

$$\frac{x - 3}{2(x - 5)} + \frac{10}{-1(x^2 - 25)} = 1$$

Then we can use as our LCD the expression

$$-2(x - 5)(x + 5)$$

Following from this, we obtain a quadratic equation which gives us two values. Checking these values, we should discover that this choice is not correct.

Since the LCD is $2y$ we multiply by it and get an equation, which we solve. However, we should find that the solution does not agree with this choice.

Please return to page 102/2 and try question 18 again.

If the machine produces 200 transformers in 5 days, it is working at the rate of 40 per day. Since this <u>is</u> a correct rate of work, it <u>is not</u> the correct choice.

Please return to page 129/2 and try question 2 again.

The factors of the first denominator are

$$(x - 3)(x + 3)$$

while the factors of the second denominator are

$$(x - 3)(x - 1)$$

Then the LCD is $(x - 3)(x + 3)(x - 1)$

Multiplying both sides by the LCD, we get

$$\begin{aligned}
2x(x - 1) + 1(x + 3) &= 2(x - 3)(x + 3) && [D \\
2x^2 - 2x + x + 3 &= 2(x^2 - 9) && [D \wedge C \\
2x^2 - x + 3 &= 2x^2 - 18 && \swarrow -2x^2, -3 \\
-x &= -21 && \swarrow \cdot(-1) \\
x &= 21
\end{aligned}$$

Now we MUST check this value in the original equation. The arithmetic may be painful, but it is necessary. We have:

$$\frac{2(21)}{21^2 - 9} + \frac{1}{21^2 - 4(21) + 3} = \frac{2}{21 - 1} \quad [\text{But } 21^2 = 441,$$

$$\frac{42}{441 - 9} + \frac{1}{441 - 84 + 3} = \frac{2}{20}$$

$$\frac{42}{432} + \frac{1}{360} = \frac{1}{10} \quad \left[\frac{42}{432} = \frac{7 \cdot 6}{72 \cdot 6}\right.$$

$$\frac{7}{72} + \frac{1}{360} = \frac{1}{10} \quad \left.\frac{7 \cdot 5}{72 \cdot 5} = \frac{35}{360}\right.$$

$$\frac{35 + 1}{360} = \frac{1}{10}$$

$$\frac{36}{360} = \frac{1}{10}$$

Finally, we see that this value does check. Since the solution is 21 this choice is correct.

Please proceed to question 17 on page $\frac{109}{1}$ after you have recovered your strength.

Question 17

Apply your knowledge to solve the equation,

$$\frac{2}{m} + \frac{1}{5} = 1$$

Which statement about the value of m is correct?

(A) $m < 1$

(B) $1 < m < 3$

(C) $3 < m < 5$

(D) $m > 5$

The question asked for a rate of work.

This requires the statement of what part of the job would be completed in one unit of time.

Your choice does not indicate any time period and, therefore, cannot be a rate.

Please return to page 123/2 and reconsider the question.

110
1

Multiplying by the LCD which is $2(y + 3)$ we get the equation:

$$\begin{array}{rcl} 2y(2) - 5(y + 3) = 1(2) & & [D \\ 4y - 5y - 15 = 2 & & [\text{Collect} \\ -y - 15 = 2 & & \swarrow + 15 \\ -y = 17 & & \swarrow \cdot (-1) \\ y = -17 & & \end{array}$$

Since the right side of the equation is $\frac{1}{y + 3}$, we replace y by -17 and get the result $-\frac{1}{14}$.

Please proceed to question 15 below.

.....

110
2

Question 15

Apply your knowledge to solve the equation,

$$\frac{2x}{x - 2} + \frac{3}{x + 1} = \frac{2x^2 - x}{x^2 - x - 2}$$

Which set contains the solution?

(A) {Positive numbers less than 5}
(B) {Positive numbers greater than 5}
(C) {Negative numbers}
(D) {None of these.}

$\frac{11}{1}$

Remember that the amount of work can be gotten by multiplying the rate of work by the time worked on the job. Also the units of rate of work must agree with the units of time. If the rate is expressed as items per hour, then the time must be expressed, etc.

Please return to page $\frac{135}{2}$ and try question 3 again.

$\frac{111}{2}$

If the second carpenter did $\frac{1}{3}$ of the job in a day, and we were told that the first carpenter also did $\frac{1}{3}$ per day, that would tell us that they did $\frac{2}{3}$ of the job in a day when working together. But this contradicts the problem. Then this choice is not correct.

Please return to page $\frac{118}{2}$ and try question 6 again.

112
1

According to the words of the problem, the machine produces 200 transformers in one week. Then this choice is not correct since this is a correct rate of work.

Please return to page 129 and try question 2 again.
2

112
2

The way to find the amount of work they will accomplish together, is to add the amounts of work which they do separately.

If Bob does the job in two hours, what part of the job does he accomplish in one hour?
Continue in this way, and you should arrive at the correct choice.

Please return to page 127 and try question 5 again.
2

Since the LCD is $2y$ we multiply by it, getting the equation:

$$6 - 5 = 2y$$

Solving this equation, we find that $y = \frac{1}{2}$. It is now necessary to substitute this value in the original equation, which becomes:

$$\frac{3}{\frac{1}{2}} - \frac{5}{1} = 1$$

Remember that we can simplify a complex fraction by multiplying numerator and denominator by the same quantity (in this case, we use 2). Then

$$\frac{2 \cdot 3}{2 \cdot \frac{1}{2}} = \frac{6}{1}$$

and the check is $6 - 5 = 1$

You should realize that if you begin by multiplying by too large a quantity; that is, a multiple of the denominators that is not the lowest, you may introduce extraneous roots. For example, if you multiplied by $2y^2$ the equation would be

$$6y - 5y = 2y^2$$

and there would be two solutions $\frac{1}{2}$ and 0. Of course 0 does not check since we would then have a zero denominator. The best advice, then, is to multiply by the Lowest Common Multiple of the denominators.

Please proceed to question 19 below.

.....

Question 19

Apply your knowledge to find the solution set of the equation

$$\frac{6t}{t - 2} + 1 = \frac{10t + 5}{2t^2 - 3t - 2}$$

Which is the correct statement about the solution set?

(A) There is only one solution and it is positive.

(B) There is only one solution and it is negative.

(C) There are two solutions; one positive and one negative.

(D) The solution set is $\emptyset$.

$\frac{114}{1}$

While it is true that we cannot determine every bit of information about the problem such as the number of exam papers, we can determine a number which is the rate of work.

Please return to page $\frac{123}{1}$ and try question 1 again.

$\frac{114}{2}$

It appears that you found the correct rate of work, which is $\frac{1}{h}$ of the job per hour. Now, since the rate of work is the amount of work done per hour, what must you do to find the amount of work done in x hours.

Please return to page $\frac{120}{2}$ and try question 4 again

It will simplify things if you observe that the denominators are in different order; one is in ascending powers and the other is in descending powers. We can take care of it as shown in the following:

$$\frac{x - 3}{2x - 10} + \frac{10}{25 - x^2} = 1$$

$$\frac{x - 3}{2(x - 5)} + \frac{10}{-1(x^2 - 25)} = 1$$

$$\frac{x - 3}{2(x - 5)} + \frac{10}{-1(x - 5)(x + 5)} = 1$$

Now we can multiply by the LCD, which is $(-2)(x - 5)(x + 5)$.

After simplification, the resulting equation is:

$$x^2 - 2x - 15 = 0$$

And solving it gives us the values 5 and -3 .

Since $x = 5$ makes the denominators zero, it is not a solution.

Using $x = -3$ we get

$$\frac{-6}{-16} + \frac{10}{16} = 1$$

Then this value checks, and this choice is correct.

You have now completed this segment.

You should now do ASSIGNMENT 12 problems 9 - 12 .

116
1

Did you begin by finding the part of the job done for each reaper? That would give you the amount each one does per hour. Now all you need do is find the total amount accomplished per hour when both are working. You can then calculate the amount done in two hours. Follow these steps carefully, and you will find that one of the other choices is correct.

Please return to page 136
2
and try question 7 again.

.....

116
2

It should be clear that this equation gives us the value 18 for x . But, if one pipe can fill the tank in 10 hours, would it take longer when a second pipe is used in addition? Obviously this is wrong and, therefore, this choice is not correct. Try to find rates of work and proceed from there.

Please return to page 149
2
and try question 10 again.

If we regard the job as the production of 200 transformers, then the whole job was performed in five days. Consequently, the rate of work is one-fifth of a job per day. Since this choice is a correct rate of work, it is not the correct choice for the problem.

Please return to page 129/2 and try question 2 again.

.....

First find the amount each typist produces per day. Then add the parts of the work done in one day when all of them are working.
After that, you should be able to figure out the amount that they would produce in two days. When you do all the calculations correctly, you will find that this choice is not correct.

Please return to page 121/2 and try question 8 again.

118
1

We begin by finding the rate of work for each of the boys.

Bob's rate of work is $\frac{1}{2}$ of a job per hour while Al's rate is $\frac{1}{3}$ of a job per hour.

I

	Hours for Complete Job	Part Done In One hour
Bob	2	$\frac{1}{2}$
Al	3	$\frac{1}{3}$

II

The amount of work done in one hour by both boys working together is the sum of the two parts of the job done by each in that hour.

Please proceed to question 6 below.

118
2

Question 6

Working together, two carpenters can do one-half of an attic expansion in a day. If one of the carpenters could do one-third of the job alone in a day, apply your knowledge to find how much work the second carpenter does in a day.

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) None of these.

The factors of the second denominator are

$$(t - 2)(2t + 1)$$

Then this is the LCD and multiplying by it gives us an equation with no fractions. After simplifying, the equation is:

$$14t^2 - 7t - 7 = 0$$

After dividing by 7, we factor and find the two values of t to be $-\frac{1}{2}$ and 1. Now we check.

If $t = -\frac{1}{2}$

the second denominator has the value zero, which is impossible.

If $t = 1$

the evaluation is as follows:

$$\frac{6}{1 - 2} + 1 = \frac{10 + 5}{2 - 3 - 2}$$

$$\frac{6}{-1} + 1 = \frac{15}{-3}$$

$$-6 + 1 = -5$$

Then the only correct solution is

$$t = 1$$

Please proceed to question 20 below.

Question 20

Apply your knowledge to find the solution set of the equation:

$$\frac{x - 3}{2x - 10} + \frac{10}{25 - x^2} = 1$$

Which statement about the solution set is correct?

(A) There is only one solution and it is positive.

(B) There is only one solution and it is negative.

(C) There are two solutions; one positive and one negative.

(D) None of the other statements is correct.

Since the machine requires 2 hours or 120 minutes to do a job, its rate of work is $\frac{1}{120}$ of a job per minute.

Then, after working for 30 minutes, the machine has performed $\frac{30}{120}$ of the job. Then the amount of work done is expressed as the fraction $\frac{1}{4}$ which is equivalent to this.

Please proceed to question 4 below.

Question 4

If a pump takes h hours to empty a tank, choose the amount of work it will do in x hours, when x is smaller than h .

(A) $\frac{x}{h}$

(B) $\frac{h}{x}$

(C) xh

(D) $\frac{1}{xh}$

Let's review the procedure.

I The problem states that one reaper can do the job in 5 hours and the other can do the same job in 8 hours.
We find what part of the job is done by each in 1 hour and then the part done by each in two hours.
We first arrange these factors in chart form.

	Hours to do Job	Part Done In 1 Hour	Part Done in 2 Hours
Reaper A	5	$\frac{1}{5}$	$\frac{2}{5}$
Reaper B	8	$\frac{1}{8}$	$\frac{2}{8}$
Both			x

II To find how much of the job is completed in two hours by both, we have to add their individual accomplishments.

$$x = \frac{2}{5} + \frac{2}{8} \quad \text{[Reduce}$$

$$x = \frac{2}{5} + \frac{1}{4} \quad \text{[Convert to like fractions.}$$

$$x = \frac{8}{20} + \frac{5}{20} \quad \text{[Add}$$

$$x = \frac{13}{20}$$

Now proceed to Question 8 below.

Question 8

One typist would take 5 days to prepare a certain report. A second typist would require 8 days, while a third would need 10 days. If all of them worked for two days, apply your knowledge to find how much work they would accomplish.

(A) $\frac{17}{40}$ (B) $\frac{2}{23}$

(C) $\frac{6}{23}$ (D) None of these.

VOLUME 12 SEGMENT 4 BEGINS HERE:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS:	48 and 50	1	4	(Sequence Number)
	54 and 56	0	4	(Type of Punch Card)
	60 and 62	1	2	(Volume Number)
	66 and 68	0	4	(Segment Number)

Your reading ASSIGNMENT for this Segment is page 314 - 315 .

SUPPLEMENTARY NOTES

In a work problem, the tasks may be the printing of 1000 books, or it may be the painting of one building, but there is always a total task. The rate of work for one person (or machine) is that fraction of the total job which the person can do in a unit of time. For example, if a house painter can paint the trim on a certain house in 6 hours, his rate of work is $\frac{1}{6}$ of the job per hour. It might take him three days to paint the entire exterior of the house; then his rate of work would be $\frac{1}{3}$ of the job per day.

We must be careful to avoid confusion between the time a man works on a certain job and the time it would take him to do the entire job by himself.

Please go on to page 123.
1

If a man can complete a job in d days by himself, then he does $\frac{1}{d}$ of the job in one day. If he works for x days, then he will complete $x \cdot \frac{1}{d}$ of the job.

There is another basic principle that is fundamental to work problems. It is assumed that if one person does $\frac{1}{a}$ of a job in a unit of time and a second does $\frac{1}{b}$ of the same job in the same unit of time then working together they will accomplish $(\frac{1}{a} + \frac{1}{b})$ of the job in the same unit of time.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

If Mr. R can grade all his final exam papers in 5 hours, which do you recognize as his rate of work?

(A) $\frac{1}{5}$ of a paper per hour

(B) $\frac{1}{5}$ of the number of final exam papers

(C) $\frac{1}{5}$ of the job per hour

(D) It cannot be determined.

$\frac{124}{1}$

Let us try to check this choice.

If the second carpenter does $\frac{1}{4}$ of the job per day, and the first one does $\frac{1}{3}$ of the job per day, then their combined work would be the total of the two fractions, which is $\frac{7}{12}$. But this contradicts the problem; and, therefore, this choice is not correct.

Please return to page $\frac{118}{2}$ and try question 6 again.

$\frac{124}{2}$

One method used to disprove an answer choice is to accept it, calculate with it and then to arrive at a contradiction with the conditions of the problem.

If this choice were correct, we would have two presses working together where each need 3 hours to do the job alone. Then together, each would do half of the job and they would require one-half of the time, or $1\frac{1}{2}$ hours to do their share.

Since this disagrees with the problem which states that together they require 2 hours, this choice is not correct.

Please return to page $\frac{152}{2}$ and try question 12 again.

Solving this equation gives us $(x = 9)$. But if the second pipe can fill the tank alone in 8 hours, will it take longer when the first pipe is used in addition? Obviously not. If you will first find the rates of work, it should be possible to proceed correctly.

Please return to page 149/2 and try question 10 again.

I The hourly accomplishment for the master bricklayer is $\frac{1}{12}$ of the job and that of the apprentice is $\frac{1}{18}$ of the job. If x represents the amount of time needed to complete the job, we get:

	Part Done in One Hour	Number of Hours on Job	=	Amount of Work Done
Master	$\frac{1}{12}$	$3 + x$		$\frac{3 + x}{12}$
Apprentice	$\frac{1}{18}$	x		$\frac{x}{18}$

II Form the equation by considering that the sum of the amount done by each equals the complete job.

If you solve this equation correctly, you will find that this choice is not correct.

Please return to page 150/1 and try question 15 again.

126/1

Remember one method is to consider the completed job as represented by 1; that is, m minutes times the part done on one minute, which is $\frac{1}{m}$. If the times for the two machines are respectively 25 minutes and 40 minutes, their rates of work are $\frac{1}{25}$ and $\frac{1}{40}$ of the job per minute. Then in m minutes, the amounts of work produced would be $\frac{m}{25}$ and $\frac{m}{40}$. But we are told that they complete the job in this time. Then the equation is:

$$\frac{m}{25} + \frac{m}{40} = 1$$

Solving this equation will give you a value which does not agree with this choice.

Please return to page 143/2 and try question 13 again.

126/2

This problem is a switch. Here we don't know what the time performance of each machine working alone is; just that one works twice as long as the other to get the job done.

	Hours to Complete Jobs Alone	In One Hour	Hours on Job	=	Part Work Done
Old	2x	$\frac{1}{2x}$	4		$4(\frac{1}{2x})$
New	x	$\frac{1}{x}$	4		$4(\frac{1}{x})$

The equation is: $\frac{2}{x} + \frac{4}{x} = 1$

When we solve this equation for x we find that this choice is not correct.

Please return to page 148/2 and try question 17 again.

Since the pump takes h hours to do the whole job, its rate of work is $\frac{1}{h}$ of the job per hour. Then in x hours, it will do x times as much as its rate of work for 1 hour. Therefore, this choice is correct.

It is important to note the restriction the problem gives for x. If x were equal to h the entire job would be finished, and if x were larger than h, the tank would have been pumped dry and still the pump would continue working. With x smaller than h we know that the pump has accomplished something while it has been operating.

Please proceed to question 5 below.

Question 5

Bob can mow a certain lawn in 2 hours, while Al needs 3 hours for the job. If they work together for 1 hour, apply your knowledge to find the amount of work they will accomplish.

(A) $\frac{1}{5}$

(B) $\frac{2}{5}$

(C) $\frac{1}{3}$

(D) $\frac{5}{6}$

128
1

The work accomplished by each typist in one day is respectively

$$\frac{1}{5} \text{ , } \frac{1}{8} \text{ , and } \frac{1}{10}$$

Then the amount that they would produce in one day while working together is

$$\frac{1}{5} + \frac{1}{8} + \frac{1}{10}$$

which equals

$$\frac{17}{40}$$

But, we were asked to find the amount of work they would do in two days.

Then the correct value is

$$\frac{34}{40} \text{ or } \frac{17}{20}$$

Since none of the other choices offers this value, this choice is correct.

Please proceed to question 9 below.

128
2

Question 9

If one man does $\frac{a}{x}$ part of a job and a second man does $\frac{b}{y}$ part of the job, apply your knowledge to write an equation which expresses the fact that they have completed the whole job.

(A) $\frac{a}{x} = \frac{b}{y}$

(B) $\frac{a}{x} + \frac{b}{y} = 1$

(C) $\frac{x}{a} + \frac{y}{b} = 1$

(D) None of these

Since it would take five hours to do the entire job; that is "grade all his final exam papers," he is working at the rate of one-fifth of the job per hour.

Then this choice is correct.

We are not concerned with the number of papers or with the length of the individual papers.

Please proceed to question 2 below.

Question 2

A machine produces 200 transformers in one week of 5 eight-hour days. Which do you recognize is NOT a correct rate of work?

(A) 40 transformers per day

(B) 25 transformers per hour

(C) 200 transformers per week

(D) $\frac{1}{5}$ of a job per day

130
1

Since the amount they do together in one day is the sum of their individual amounts of work, it follows that we can calculate any one of those items if we know the other two. But that is exactly the situation here. Then we can calculate the required answer, and we should find that one of the other choices lists it.

Please return to page 118/2 and try question 6 again.

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130
2

One method used to disprove an answer choice is to accept it, calculate with it; and then arrive at a contradiction with the conditions of the problem. If the first press needs 3 hours alone to complete the job and the second press needs 4 hours, we can check the problem by finding the amount of time they would need together. The accomplishments of each per hour are $\frac{1}{3}$ and $\frac{1}{4}$ of the job respectively, which gives us a total of $\frac{7}{12}$. That would mean that together the two presses print $\frac{7}{12}$ of the edition per hour. But the original problem said that the two presses require 2 hours, which means that they do one-half of the job per hour. Since our figures ($\frac{7}{12}$) does not agree with the figure in the original problem ($\frac{1}{2}$), this choice is not correct.

Please return to page 152/2 and try question 12 again.

You were on the right track, but you went astray. The part of the job done in one hour by each of two pipes is $\frac{1}{8}$ and $-\frac{1}{10}$. But if you add two parts done, you will get the total amount of work done in one hour, not the time it will take together. Whenever you add quantities, the units must be the same and the total will have the same units. For example, if you add two figures in amount of work per day, the result is an amount of work per day.

Please return to page $\frac{149}{2}$ and try question 10 again.

.....

The formula which gives the distance is

$$\text{rate} \cdot \text{time} = \text{distance}$$

In using this formula, it is necessary to have the correct units for the different quantities. If you apply the formula correctly, you will discover that this choice is not correct.

Please return to page $\frac{160}{2}$ and try question 1 again.

Although we don't know the actual time it takes each machine to complete the job alone we do know that one takes twice as long as the other, and that together they finish in 4 hours. Therefore, arranging our information, we have:

	Hours to Complete Job Alone	Part of Job Done in One Hour	Hours on Job	=	Part of Work Done in Four Hours
Old	$2x$	$\frac{1}{2x}$	4		$\frac{4}{2x}$
New	x	$\frac{1}{x}$	4		$\frac{4}{x}$

Note that the old pump would need $2x$ hours alone, so that its rate of work is $\frac{1}{2x}$. Also the fraction $\frac{4}{2x}$ reduces to $\frac{2}{x}$. Then the equation is:

$$\frac{2}{x} + \frac{4}{x} = 1$$

We multiply by the LCD, x

$$2 + 4 = x$$

$$6 = x$$

Then this choice is correct.

Checking this solution, we see that the old pump needs 12 hours alone, giving it a rate of work of $\frac{1}{12}$ of the job per hour. Then in 4 hours, it does $\frac{1}{3}$ of the job. The rate of work of the new pump is $\frac{1}{6}$ of the job per hour, so that the amount of work it performs in 4 hours is $\frac{2}{3}$ of the job. Then together, the two machines have done one whole job.

Please proceed to question 18 below.

.....

Question 18

A paperhanger can paper a wall in 8 minutes less than his assistant. Working together, it takes them $7\frac{1}{2}$ minutes to paper the wall. Apply your knowledge to find y, the number of minutes it would take the paperhanger to do the job by himself. Which statement about the value of y is correct?

(A) $y \leq 5$

(B) $5 < y < 10$

(C) $10 < y \leq 15$

(D) $y > 15$

That's right.

Given the times for the machines to complete the job individually as 25 and 40 minutes respectively, their accomplishments are

$$\frac{1}{25} \text{ and } \frac{1}{40}$$

of the job in 1 minute.

Then in m minutes, the amounts of work produced are

$$\frac{m}{25} \text{ and } \frac{m}{40}$$

Using the fact that the total job is completed, we get the equation:

$$\frac{m}{25} + \frac{m}{40} = 1 \qquad \propto (200) \text{ (the LCD)}$$

$$8m + 5m = 200 \qquad \propto \text{Combine}$$

$$13m = 200 \qquad \propto \div 13$$

$$m = 15\frac{5}{13}$$

and $25 > m > 15$

Then this choice is correct.

Please proceed to question 14 below.

.....

Question 14

Working together, a farmer and his son can plow a field in 6 hours. If the farmer, working alone, would take 8 hours, how long would it take the son alone to plow the field? Apply your knowledge to find the set containing the correct answer.

(A) $\{10, 16, 20\}$
(B) $\{18, 24, 30\}$
(C) $\{14, 21, 28\}$
(D) $\{2\}$

134
1

This choice states that the amount of work done by each man is the same. But the problem did not say that. How would you represent the fact that they completed the whole job?

Please return to page 128 (2) and try question 9 again.

134
2

It appears that you have multiplied rate times time in order to get distance. That is correct in general, but what rate did you use? The problem states that the airplane is flying against the wind; therefore, it is necessary to use the rate of flying against the wind. What is that rate? The problem neglects to state it.

Please return to page 146 (2) and try question 2 again.

In one week of five eight-hour days, there are 40 hours.

Then 200 transformers in 40 hours is not equivalent to a rate of 25 transformers per hour. Since this is not a correct rate of work, this choice is correct.

Please proceed to question 3 below.

Question 3

Perform the calculation to find the amount of work done in 30 minutes by a machine which requires 2 hours to do a job.

(A) $\frac{1}{15}$

(B) 15

(C) $\frac{1}{4}$

(D) None of these.

136

We are told that the combined amount of work accomplished by both carpenters in one day is $\frac{1}{2}$ of the job, while one can complete $\frac{1}{3}$ of the job in that time by himself.

This information can be arranged in chart form.

		Part Done in 1 day.
I	A	$\frac{1}{3}$
	B	x
Together		$\frac{1}{2}$

II The contribution of one can be found by subtracting the work done by the other from the total amount done by both.

$$x = \frac{1}{2} - \frac{1}{3} \qquad [\text{ Convert to LCD}$$

$$x = \frac{3}{6} - \frac{2}{6}$$

$$x = \frac{1}{6}$$

136

Question 7

If one reaper can harvest a field in 5 hours, while a second reaper would take 8 hours, apply your knowledge to find the amount that would be harvested in 2 hours if both worked together.

(A) $\frac{13}{20}$ (B) $\frac{2}{13}$

(C) $\frac{4}{13}$ (D) None of these.

One method to disprove an answer choice is to accept it, calculate with it and then arrive at a contradiction with the conditions of the problem. The problem states that the two presses require 2 hours together; then their combined rate of work is one-half of the job per hour. Using the figure of this choice, the separate accomplishments are $\frac{1}{3}$ and $\frac{1}{8}$ of the job per hour. The total of these is $\frac{11}{24}$ which does not equal the stated rate of $\frac{1}{2}$.

Then this choice is not correct.

Please return to page 152/2 and try question 12 again.

Did you use the formula:

rate . time = distance ?

If it is used properly, it will give you the correct answer, which is offered in one of the other choices.

Please return to page 160/2 and try question 1 again.

$\frac{138}{1}$

Did you set up the equation properly?

Let's review the beginning.

If we let s represent the time it would take the son to plow the field alone, his one hour accomplishment would be $\frac{1}{s}$ of the field. The farmer takes 8 hours alone so that he does $\frac{1}{8}$ of the field in one hour. Since they can do the job together in 6 hours, we know that together they plow $\frac{1}{6}$ of the field in one hour. The sum of the individual accomplishments for one hour equals the amount of both.

$$\frac{1}{s} + \frac{1}{8} = \frac{1}{6}$$

If you solve this correctly, you will find that this choice is not correct.

Please return to page $\frac{133}{2}$ and try question 14 again.

$\frac{138}{2}$

The formula for this problem is:

$$\text{rate} \cdot \text{time} = \text{distance}$$

Since the units attached to the quantities in this problem fit together, we can substitute them in the formula. However, either you have substituted incorrectly, or you made an error in algebra.

Please return to page $\frac{163}{2}$ and try question 4 again.

If it takes d days to complete a job; in one day $\frac{1}{d}$ of the job is done. Now if this amount is done each day for d days, there will be

$$d \times \frac{1}{d}$$

of the job done. Therefore, it is correct to represent the complete job as 1 but what have you added to get the total? This choice is not correct.

Please return to page $\frac{128}{2}$ and try question 9 again.

In order to find the rate of flying against the wind, it is necessary to perform a subtraction. But the subtraction in this choice is impossible, since the 550 is in miles per hour while the h is in hours. Addition and subtraction can be performed only when both quantities are in the same units. In any case, a subtraction of rates would not give a distance, and you were looking for a distance.

Please return to page $\frac{146}{2}$ and try question 2 again.

$\frac{140}{1}$

That's right.

If the first pipe takes 10 hours to fill the tank, its rate of work is $\frac{1}{10}$ of the job in one hour. Similarly the rate of work of the second pipe is $\frac{1}{8}$ of the job in one hour. Then if each pipe is working for x hours; the amounts of work they do are respectively $\frac{x}{8}$ and $\frac{x}{10}$. Since the full tank is 1 job, the equation should state that their total is 1. That is exactly what this equation tells us, therefore, this choice is correct.

There is a second method for handling this type of problem.
We start in the same way:

	Number of Hours to Fill Tank	Part of Job Done in One Hour
First Pipe	10	$\frac{1}{10}$
Second Pipe	8	$\frac{1}{8}$

But we consider both working together as if they were a third pipe and the number of hours to finish as x.

Together	x	$\frac{1}{x}$

Since the part done in one hour by both is the total of the parts done by each, we can write the equation

$$\frac{1}{10} + \frac{1}{8} = \frac{1}{x}$$

Now, if we multiply both sides of the equation by x we get

$$\frac{x}{10} + \frac{x}{8} = 1$$

This then is another explanation of the statement that the complete job can be represented by 1.

Please proceed to page $\frac{141}{1}$.

Question 11

Apply your knowledge to solve the equation

$$\frac{x}{5} + \frac{x}{10} = 1$$

Which domain contains the single value you found for x ?

(A) $x < 4$

(B) $4 < x < 6$

(C) $6 < x < 8$

(D) $x > 8$

It is true that this problem takes place in a stream where the rate of the current should be considered. However, if you consider the wording of the problem, you will discover that the rate of the current is not important in answering this question. Then this choice is not correct.

Please return to page 158/2 and try question 3 again.

142/1

It looks like you need some assistance in this one.

I The information can be put in this form:

	Hourly Rate of Work	•	Hours On Job	=	Part of Work Done
Fill	$\frac{1}{6}$		$2 + n$		$\frac{2 + n}{6}$
Drain	$-\frac{1}{9}$		n		$-\frac{n}{9}$

Note: The drain must have the opposite sign to the sign of the fill

II Now set up the equation combining the effects of the two pipes to equal the complete job.

Solving the equation, we find that this choice is not correct.

Please return to page 157/1 and try question 16 again.

142/2

This is a difficult problem. Let's try to organize our information.

	Minutes to Complete Job Alone	Part of Job Done in 1 minute	• Number of Minutes Worked	= Part of Job Done
Paperhanger	y	$\frac{1}{y}$	7.5	$7.5\left(\frac{1}{y}\right)$
Assistant	$y + 8$	$\frac{1}{y + 8}$	7.5	$7.5\left(\frac{1}{y + 8}\right)$

Now we can see that if it takes the paperhanger y minutes by himself, then it takes his assistant $y + 8$ minutes. Since each works 7.5 minutes, the entire job is done. Remember, the entire job can be represented by 1. Therefore, we get the equation:

$$\frac{7.5}{y + 8} + \frac{7.5}{y} = 1$$

When we solve this equation, we get two different values for y, but neither of them agrees with this choice.

Please return to page 132/2 and try question 18 again.

We can check your answer by assuming that it's true and seeing whether the relationships in the problem are satisfied to do the job alone.
If one press requires 3 hours to do the job alone and the other needs 6 hours, their respective accomplishments are $\frac{1}{3}$ and $\frac{1}{6}$ of the job in one hour.

Then working together for one hour, they would accomplish the sum of these figures in one hour. But the sum of these fractions is $\frac{3}{6}$ or $\frac{1}{2}$. Then they would do one-half the job in one hour, or the whole job in two hours. The problem could also be analyzed in this manner, directly in chart form:

	Hours to Complete	Part Done In 1 Hour
Press A	3	$\frac{1}{3}$
Press B	x	$\frac{1}{x}$
Both	2	$\frac{1}{2}$

$$\frac{1}{3} + \frac{1}{x} = \frac{1}{2} \qquad \text{(6x)}$$

$$2x + 6 = 3x \qquad - 2x$$

$$6 = x$$

This is the information stated in the problem. Therefore, this choice is correct. Please proceed to question 13 below.

Question 13

One machine does a job in 25 minutes, but a second machine needs 40 minutes for the job. Apply your knowledge to choose the domain that contains the value of m the number of minutes it would take the two machines working together to do the complete job.

(A) $m > 40$

(B) $40 > m > 25$

(C) $m < 15$

(D) $25 > m > 15$

144
1

In order to express the fact that they have completed the whole job, it is necessary to add the amounts of work they have done separately to get the total amount of work completed. One of the other choices does offer you a correct equation.

Please return to page 128/2 and try question 9 again.

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144
2

The formula for this problem is:

rate • time = distance

If you substitute the values in this formula and solve the resulting equation for time, you will discover that one of the other choices is correct.

Please return to page 163/2 and try question 4 again.

Sometimes a quick answer seems to be right because we consider only part of the problem.
It seems that that is what happened when you made this choice.
Did you reason this way?

The farmer alone takes 8 hours, but when he is helped by his son, they take only six hours. It appears that his son helped by reducing the time by two hours. But does that mean that the son can do the job by himself in two hours?
Of course not, but that is the "quick answer" you made.

Now return to page 133/2 and give the problem the proper amount of thought

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In problems of this type, it is wise to organize the facts in chart form in order to see the relationships involved. Since this is a motion problem, we use the formula:

Rate x Time = Distance

In chart form this problem would appear as

	R	x	T	=	D
No Wind	p		?		
Against Wind	p - y		?		d

Note: The rate against the wind is found by subtracting the rate of the wind from the rate of the plane in still air.

The time can be found by using the variation of the formula $R \times T = D$ when it is solved for T, $T = \frac{D}{R}$
Now, check your work and reconsider the problem.

Please return to page 171/2.

146
1

The formula for this type of problem is:

rate • time = distance

Of course, the units must be properly selected. In this case, the rate is in miles per hour, and therefore, the time must be in hours and the distance in miles. Since that is exactly the situation we are given, this choice is correct.

Please proceed to question 2 below.

146
2

Question 2

An airplane which can fly at 550 miles per hour in still air, flies against the wind for h hours. Which choice do you recognize as the number of miles it travels?

(A) 550h

(B) 550 - h

(C) 550 + h

(D) none of these

The simplest way to solve this equation is to multiply by the LCD which is 10 . This will give us an equation with no fractions. Solving it, we discover that this choice is not correct.

Choose the domain that contains the value you found by referring to the number line concept.

Please return to page 141/1 and try question 11 again.

This problem takes place in a stream, and it is true that we are given no information on the rate of the current. However, you will discover that such information is not needed in order to answer this question. Therefore, this choice is not correct.

Please return to page 158/2 and try question 3 again.

$\frac{148}{1}$

Good, this one had a new wrinkle.

I We handle the fill and the drain by using a positive number for the "fill" and negative for the "drain."
Arranging the information in a table, we have:

	Done in One Hour	Hours on Job	=	Part of Work Done
Fill	$\frac{1}{6}$	$2 + n$		$\frac{2 + n}{6}$
Drain	$-\frac{1}{9}$	n		$-\frac{n}{9}$

II The Algebraic sum of the two effects equals the complete filling of the tank. The equation is:

$$\frac{2 + n}{6} - \frac{n}{9} = 1 \qquad \text{We multiply by the LCD, 18}$$

$$3(2 + n) - 2(n) = 18 \qquad \text{D}$$

$$6 + 3n - 2n = 18 \qquad \text{C}$$

$$6 + n = 18 \qquad -6$$

$$n = 12$$

Then this choice is correct.

In checking, we see that the fill pipe works for 14 hours, filling $\frac{14}{6}$ or $\frac{7}{3}$ of the tank. Of course this is more than full, but don't be upset! Meanwhile the drain is emptying (minus) for 12 hours, which removes $\frac{12}{9}$ or $\frac{4}{3}$ of a tank. The amount of water which has entered the tank, but has not left through the drain is $\frac{7}{3} - \frac{4}{3}$ or 1 full tank.

Please proceed to question 17 below.

.....

$\frac{148}{2}$

Question 17

An old pump needs twice as much time to complete a task as a new model. Working together, the two pumps can do the job in 4 hours. Apply your knowledge to find x, the number of hours it would take the new model to do the job alone. Which is the correct domain that contains the value of x?

(A) $4 < x < 5$

(B) $5 < x < 7$

(C) $7 < x < 10$

(D) $10 < x < 14$

It it takes d days to complete a job, then in one day, $\frac{1}{d}$ of the job is done. If they worked d days and accomplished $\frac{1}{d}$ each day, then they would accomplish

$$d \cdot \frac{1}{d} \text{ or } (1) \text{ in } d \text{ days}$$

Since they have completed the whole job, the total amount of work done is 1 . Then adding the separate amounts of work should give us 1 . Since that is exactly what this equation says, this choice is correct.

Please proceed to question 10 below.

Question 10

One pipe can fill a tank in 10 hours, but a second pipe would take only 8 hours. Apply your knowledge to find the equation which could be used to find x ; the time it will take to fill the tank using both pipes together.

(A) $10 + 8 = x$

(B) $10 + 8 = 2x$

(C) $\frac{x}{10} + \frac{x}{8} = 1$

(D) $\frac{1}{8} + \frac{1}{10} = x$

150
1

Good. Let's briefly review the analysis of the problem.

If we let s represent the time it takes the son alone, his rate of work is $\frac{1}{s}$ of the job in one hour. But the farmer's rate of work is $\frac{1}{8}$ of the job in one hour. Since they do the whole job together in 6 hours, in one hour they complete $\frac{1}{6}$ of the job. Then the equation is:

$$\frac{1}{8} + \frac{1}{s} = \frac{1}{6} \qquad \text{Multiply by the LCD, } 24s$$

$$3s + 24 = 4s \qquad -3s$$

$$24 = s$$

Then this choice is correct.

Please proceed to question 15 below.

150
1

Question 15

A master bricklayer can build a wall in 12 hours, while his apprentice would require 18 hours. The master bricklayer begins the job alone and after 3 hours he is joined by the apprentice. How long will it now take to complete the wall?

Apply your knowledge to find the correct statement about the answer.

(A) It will take them less than 6 hours.

(B) It will take them more than 6 but less than 7 hours.

(C) It will take them more than 7 but less than 9 hours.

(D) It will take them more than 9 hours.

Why did you add the 550 and the $\underline{h}$? Even if you have a good reason this addition is impossible. Only quantities involving the same units can be added and these are in different units. In addition to that, what happened to the formula for finding distance? You do need to use the formula:

$$\text{Rate} \times \text{Time} = \text{Distance}$$

Please return to page 146/2 and try question 2 again.

Since we are asked to find the time, we should change the usual basic motion formula from the form giving distance

$$\text{Rate} \times \text{Time} = \text{Distance}$$

to a form in which we find the time.

The formula becomes:

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

Since the trip involves flying against the wind, we need the rate against the wind. Did you find that?

This choice is not correct.

Please return to page 171/2 and try question 6 again.

152
1

The simplest way to solve this equation is to multiply by the LCD, which is 10 . Then we have:

$$\frac{x}{5} + \frac{x}{10} = 1 \qquad \times (10)$$

$$2x + x = 10 \qquad [\ C$$

$$3x = 10 \qquad \div 3$$

$$x = \frac{10}{3}$$

Since $\frac{10}{3} < 4$ this choice is correct.

Please proceed to question 12 below.

152
2

Question 12

Two presses working together can print one edition of a newspaper in 2 hours. If one press would take 3 hours alone, find the time the second press would need to print the edition alone. Perform the calculation to find which choice is correct.

(A) 3

(B) 6

(C) 4

(D) 8

The formula for this problem is:

$$\text{rate} \cdot \text{time} = \text{distance}$$

Then we divide both sides of the equation by rate, and we get:

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

We must check the units; with rate in miles per hour it is necessary that distance be in miles and time in hours. Since these conditions are satisfied, we substitute our values and find that this choice is correct.

Please proceed to question 5 below.

Question 5

A motorboat travels in a stream which has a 2mph current. If the motorboat travels 8 mph in still water, choose the rate at which it travels downstream.

(A) 10 mph

(B) 6 mph

(C) 8 mph

(D) None of these.

Let's organize the given information into chart form.

	Minutes to Complete Job Alone	Part of Job Done in 1 Minute	Number of Minutes Worked	Part of Job done
Paperhanger	y	$\frac{1}{y}$	7.5	$7.5(\frac{1}{y})$
Assistant	$y + 8$	$\frac{1}{y+8}$	7.5	$7.5(\frac{1}{y+8})$

If the paperhanger takes y minutes by himself, then his assistant needs $y + 8$ minutes. Since each one works for 7.5 minutes, we get the equation:

$$\frac{7.5}{y} + \frac{7.5}{y+8} = 1$$ ⟨ We multiply by the LCD, $y(y+8)$

$$7.5(y+8) + 7.5 = y(y+8)$$ [D

$$7.5y + 60 + 7.5y = y^2 + 8y$$ [Combine

$$60 + 15y = y^2 + 8y$$ ⟨ $-60, -15y$

$$0 = y^2 - 7y - 60$$ [Factor

$$0 = (y-12)(y+5)$$

Then we get two values for y ; 12 and -5 . Since the -5 has no meaning, our result is that the paperhanger needs 12 minutes, and the assistant needs 20 minutes. Therefore, this choice is correct.
We check as follows: the paperhanger does $\frac{7.5}{12}$ of the job and the assistant does $\frac{7.5}{20}$ of the job. These fractions can be simplified by multiplying numerator and denominator by 10 and then reducing. Then we have:

$$\frac{7.5}{12} = \frac{75}{120} = \frac{5}{8}$$

and

$$\frac{7.5}{20} = \frac{75}{200} = \frac{3}{8}$$

Since the total is 1 job, this completes the check. Since

$$\frac{5}{8} + \frac{3}{8} = 1$$

You have now completed this segment. Hand in the PUNCH CARD. You should enter in your NOTEBOOK the definition and formula below:

Definition: Rate of work $= \frac{1}{\text{time it takes to do job alone}}$

Formula: Rate of work $\cdot$ time on job = part of work done

You should now be able to complete ASSIGNMENT 12 , Problems 13 - 16 .

The basic formula is:

Rate Upstream = Rate in Still Water - Rate of Current

If you substitute the given items correctly and solve for the quantity you want, you should find that this choice is not correct.

Please return to page 164/2 and try question 7 again.

Your choice of variables appears to be correct because you have represented the two times correctly.
It is true that

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

The problem states that one time is 2 hours less than the other. This is the statement that indicates how you should create the equation. However, your choice states that two quantities are equal.

This choice is not correct.

Please return to page 179/2 and try question 9 again.

Good. Would you like to check your work against ours?
We handled the unequal number of hours by letting one work for x hours, while the other worked $(3 + x)$ hours.

I The rate of work for the master bricklayer is $\frac{1}{12}$ of the job per hour and that for his apprentice is $\frac{1}{18}$ of the job per hour. If we let x represent the number of hours needed to complete the job, we have:

	Part of Job Done In One Hour	· Number of Hours on Job	= Amount of Work Done
Master	$\frac{1}{12}$	$3 + x$	$\frac{3 + x}{12}$
Apprentice	$\frac{1}{18}$	x	$\frac{x}{18}$

II The sum of the individual contribution equals the total job. And the equation is

$$\frac{3 + x}{12} + \frac{x}{18} = 1 \quad \text{We multiply by the LCD, 36}$$

$$3(3 + x) + 2(x) = 36 \cdot 1 \quad [\text{ D}$$

$$9 + 3x + 2x = 36 \quad [\text{ C}$$

$$9 + 5x = 36 \quad -9$$

$$5x = 27 \quad \div 5$$

$$x = 5.4$$

Then this choice is correct.

III We can check this solution as follows: The master bricklayer will have worked a total of 8.4 hours, and done $\frac{8.4}{12}$ part of the job. The apprentice, in 5.4 hours had done $\frac{5.4}{18}$ part of the job. The first fraction equals $.7$ and the second equals $.3$, so that the total amount of work performed is 1 complete job.

Please proceed to question 16 on page 157/1.

Question 16

A tank can be filled by one pipe in 6 hours, while a drain pipe can empty the tank in 9 hours. After the first pipe has been operating for 2 hours, the drain pipe is opened. If n is the number of hours it will now take to fill the tank with both pipes operating, apply your knowledge to find the domain that contains the value of n.

(A) $n < 5$

(B) $5 < n < 9$

(C) $9 < n < 13$

(D) $n > 13$

It appears that you have the general idea of how to handle this problem. But do you think that the boat would travel downstream less rapidly than it would in still water? Perhaps you are confused about the meaning of "downstream." That is the direction in which the water is moving.

Please return to page 153/2 and try question 5 again.

158
1

In order to find the distance the plane flies, we need to know the rate at which it is flying. All we are told is that its rate in still air is 550 miles per hour. From this, we must subtract the rate of the wind in order to find its rate against the wind. Since we do not know the rate of the wind, we cannot find the answer to the question. Then this choice is correct. It is interesting to note that (550 h) is the distance the plane would fly with no wind; therefore, the plane would fly <u>less than</u> (550h) miles in h hours against the wind.

Proceed to question 3 below.

158
2

<u>Question 3</u>

A swimmer travels a distance of 20 miles downstream in 4 hours. Which do you recognize as his rate, in miles per hour, during the trip?

(A) 5

(B) more than 5

(C) less than 5

(D) it can't be found

I Setting up a table with the information in the problem:

	Rate	•	Time	=	Distance
With Wind	$r + 30$		$\frac{360}{r + 30}$		360
Against Wind	$r - 30$		$\frac{360}{r - 30}$		360

II But the problem states that it takes 1 hour longer against the wind. Then we can write an equation which states algebraically:

Time Against Wind = Time with Wind + 1

Solving this equation gives us a value which does not fit this choice.

Please return to page $\frac{176}{1}$ and try question 16 again.

You have written an equation which states that two quantities are equal. According to the problem, the two "times" are equal. But you have an error in your representation of one of the "times." It should be easy to spot, when you look for it. Suppose you organize your facts by arranging them in chart form:

Begin in this fashion:

	(R	•	T	=	D)
(1)	x		?		d
(2)	x + 2		?	d + 10	

Now determine the two times and re-read the problem for the relationship between them.

Please return to page $\frac{174}{2}$ and try question 8 again.

160
1

Volume 12 Segment 5 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	1 5	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	1 2	(Volume Number)
	66	and	68	0 5	(Segment Number)

Your READING ASSIGNMENT for this SEGMENT is pages: 316 - 317 .

SUPPLEMENTARY NOTES

Let us take another look at the illustration on page 317 of your text. Among the quantities which we were not given are: the speed of the wind; the speed of travel with the wind; the speed against the wind, the time it took the plane to fly 2520 miles with the wind. It is obvious that we cannot use a different variable for each of these quantities without becoming overwhelmed by the number of variables. Since the rate of the wind was the quantity we were asked to find, it is sensible to use a variable to stand for this quantity.

It then follows that we can express the rate with the wind and the rate against the wind in terms of this one variable. Notice that the red figures in the table on page 317 are entered after everything else by making use of the formula:

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

This type of problem may also be met in the setting where a boat is moving up or down a stream. A stream has a current which can be compared to the wind in our illustration. The current flows downstream; therefore, the rate downstream compares with the rate against the wind.

You will now be asked a series of questions to draw your attention to the more important points.

160
2

Question 1

If a boat is moving at m miles per hour, which do you recognize as the number of miles it will travel in t hours?

(A) $\frac{m}{t}$

(B) mt

(C) $\frac{t}{m}$

(D) None of these.

Perhaps you are not familiar with the flow of water in a stream or river. The direction in which the water moves is "downstream." According to this choice, the current in the river has no effect on the boat. That is not correct.

Please return to page 153/2 and try question 5 again.

It is true that you need the formula:

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

You appear to have applied that formula correctly.

The problem stated that the time with the wind is 2 hours less than the time against the wind. This is the source of the equation.

But how do you show that one quantity is smaller than another? For example, write an equation stating that y is 5 less than x .

The equation should be written as:

$$y = x - 5$$

Notice that the underlined part is written in different order, in English and in algebra. You should be able to write the equation correctly now.

Please return to page 179/2 and try question 9 again.

$\frac{162}{1}$

The problem does state that two quantities are equal, and that is what your choice says. However, what are the quantities in your equation? You have multiplied distance by rate, which is not what the formula calls for in this type of problem.

Suppose you organize your facts by arranging them in chart form as follows:

	R	x	T	=	D
(1)	x		?		d
(2)	x + 2		?		d + 10

Find the two times and then determine their relationship from the statements of the problem.

Please return to page $\frac{174}{2}$ and try question 8 again.

.....

$\frac{162}{2}$

Your answer can be disproved by assuming that it is true, calculating with it, and arriving at a contradiction.

If his rate in still water is 2 mph , and the current is 1 mph , the rate downstream would be 3 and the rate upstream would be 1 mph . We can use the rule:

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

Then the time downstream would be $\frac{20}{3}$ while the time upstream would be $\frac{10}{1}$ But then the times are not equal, therefore, this choice is not correct since the problem stated that the two trips took the same time.

Please return to page $\frac{177}{2}$ and try question 11 again.

Since rate time = distance, it follows that rate = distance divided by time. Using the total distance divided by the total time, we get 5 . Then this choice is correct. The fact that this trip is downstream does not affect the calculation. We can go a bit further than the problem. We now know that the rate downstream is 5 miles per hour. Therefore, the rate of the swimmer in still water is less than 5 miles per hour.

Please proceed to question 4 below.

Question 4

Flying at a rate of $x + y$ miles per hour with the wind, a plane covers w miles. Choose the correct value for the number of hours the plane required for the trip.

(A) $\frac{x + y}{w}$

(B) $w (x + y)$

(C) $\frac{w}{x + y}$

(D) None of these.

164
1

Starting with the formula:

$$\text{Rate} \times \text{Time} = \text{Distance}$$ ← Rate

we transform it into:

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

We are told that the distance is d miles. Flying against the wind, its rate would be the difference between its rate in still air and the rate of the wind; this rate is, therefore, $(p - y)$. Then the time required is

$$\text{Time} = \frac{d}{p - y}$$

and this choice is correct.

What happened to t ? It was not needed in the problem; and, therefore, we should ignore it. True, it is somewhat unusual to be given information that is extraneous (unnecessary), but you should be aware of what is needed and what is not.
We might observe that our choice of an answer is not larger than t , because it if were, the plane would have crashed from lack of fuel.

Please proceed to question 7 which follows.

.....

164
2

Question 7

Sailing upstream in a river whose current is known to be m miles per hour, a boat captain finds that his net speed is p miles per hour. Choose the number of miles per hour that the boat would travel if there were no current.

(A) $p - m$
(B) $m - p$
(C) $p + m$
(D) p

If you are confused about this problem, reread the text assignment before you try the question again. One of the other choices is correct.

You are dealing with two rates that affect the motion of the boat along the same line of action. One of two situations results. Either the two motions are in the same direction, and the net rate is the sum of the two rates; or the two motions are in directly opposite directions, and the net rate is the difference of the two rates.

Please return to page 153/2 and try question 5 again.

.....

Since the denominators cannot be factored, the LCD is equal to

$$(12 + x)(12 - x)$$

If we multiply both sides of the equation by this LCD, we get the equation:

$$15(12 - x) = 9(12 + x)$$

When you solve this equation correctly and check your answer, you will find that this choice is not correct.

Please return to page 172/2 and try question 10 again.

I VARIABLES:

If we use c to represent the rate of the current, we can organize our information as follows:

	rate	x	time	=	distance
downstream	$10 + c$				12
upstream	$10 - c$				12

Now we fill in the items missing from the time column, and we get:

$$\text{time downstream} = \frac{12}{10 + c} \quad \text{and time upstream} = \frac{12}{10 - c}$$

II RELATIONSHIP:

Since the total time is $2\frac{1}{2} = \frac{5}{2}$ we can write the equation:

$$\text{time down} + \text{time up} = \text{total time}$$

$$\frac{12}{10 + c} + \frac{12}{10 - c} = \frac{5}{2}$$

We multiply by the LCD: $2(10 + c)(10 - c)$

$$24(10 - c) + 24(10 + c) = 5(10 + c)(10 - c) \qquad [D$$

$$240 - 24c + 240 + 24c = 5(100 - c^2) \qquad [C \wedge D$$

$$480 = 500 - 5c^2 \qquad \swarrow +5c^2, -480$$

$$5c^2 = 20 \qquad \swarrow \div 5$$

$$c^2 = 4$$

$$c = 2 \text{ and } c = -2$$

Obviously $c = -2$ has no meaning. Let us check $c = 2$.

The rate downstream is 12 so that it takes 1 hour to go downstream 12 miles. The rate upstream is 8 which makes the time upstream equal to

$$\frac{12}{8} = 1\frac{1}{2}$$

Then the total time is $2\frac{1}{2}$ hours which checks. Therefore, this choice contains the rate of the current.

	R	x	T	=	D	
Down	12		(1)		12	$1 + \frac{12}{8} = 2\frac{1}{2}$
Up	8		$(\frac{12}{8})$		12	$1 + 1\frac{1}{2} = 2\frac{1}{2}$

Proceed to question 13 on page 167/1.

Question 13

The rate of a boat in still water is 12 mph . If the rate of the current is 3 mph, apply your knowledge to find how far upstream the boat can travel before returning to its starting point, when the time allowed for the entire trip is 4 hours. Which statement about the distance d is correct?

(A) d is an odd integer
(B) d is an even integer
(C) d is between 16 and 24 miles
(D) d is between 8 and 16 miles

.....

The problem states that the same time was involved in both cases. However, your choice says that one quantity is larger than the other quantity by 2 . It is true that time can be found by dividing distance by rate, but what meaning is there for the fraction $\frac{d + 10}{x}$? There are actually two different reasons why this choice is not correct.

(1) The distance in the second case, d + 10 , should not be divided by the rate of the first case, since the rate has changed. This expression is not accurate.

(2) The time in the two cases is said to be equal, but you said one time is "2" more than the other.

Please return to page 174/2 and try question 8 again.

168/1

You have it partially correct.

The time is found by dividing the distance by the rate.

Therefore, the time against the wind would be represented by

$$\frac{m}{8 - w}$$

since the distance is m and the rate against the wind is $8 - w$.

Now the rate with the wind is $8 + w$ as you indicated. However, the distance is still m, not $m + 2$ which is a mixture of distance and time.

Consider this relationship in setting up the equation:

time with wind equals time against wind less two hours.

Please return to page 179/2 and reconsider the problem.

.....

168/2

If the rate in still water is 4 mph, while the rate of the stream is 1 mph, the rate downstream is 5 and the rate upstream is 3. Then the time downstream is $\frac{20}{5}$ and the time upstream is $\frac{10}{3}$.

	R	x	T	=	D
Still	4				
Down	4 + 1		$\frac{20}{5}$		20
Up	4 - 1		$\frac{10}{3}$		10

Since these are not equal, and the problem said they should be equal, this choice is not correct.

Please return to page 177/2 and try question 11 again.

According to this choice, the current makes no difference in the rate of the boat. Is that what you meant? It is not correct.

Were you looking for some kind of trick? In the last problem you were given information that had nothing to do with the situation. But it is up to you to examine the information to see if it does belong to the problem. In this case, it belongs.

Please return to page 164/2 and try question 7 again.

.....

I If x represents the rate of the wind, we have:

	rate	·	time	=	distance
with wind	100 + x		?		600
against wind	100 - x		?		500

II Now, after finding an algebraic expression for the two times we can write an equation which states that one time is $\frac{4}{5}$ of the other. After solving the equation, we find that this choice is not correct.

Please return to page 182/2 and try question 15 again.

I Arranging our facts in a table where we fill in the items in the time column last, we have:

	rate	x	time	=	distance
with wind	$220 + w$		$\frac{480}{220 + w}$		480
against wind	$220 - w$		$\frac{400}{220 - w}$		400

II Since we were told that the times are equal, our equation is:

$$\frac{480}{220 + w} = \frac{400}{200 - w}$$

Multiplying by the LCD which is $(200 + w)(200 - w)$

$$480(220 - w) = 400(200 + w) \quad \text{[D}$$
$$105600 - 480w = 88000 + 400w \quad \text{[} -480w, -88{,}000$$
$$17600 = 880w \quad \text{[} \div 880$$
$$20 = w$$

You might note that if you divided both sides of the original equation by 80 the numerators would have reduced to 6 and 5, and the arithmetic would have been much simpler.

III Checking, we find that the rate with the wind is 240, which means it would take 2 hours to fly 480 miles. The rate against the wind is 200, so that it would take 2 hours to fly 400 miles.

	R	x	T	=	D
With	240		2		480
Against	200		2		400

Since the time is the same for both directions, this value checks.

Please proceed to question 15 on page 182/2.

Since the current is flowing downstream it would push the boat at 2 miles per hour even if the motor were not operating. When the motor pushes the boat at 8 mph and the current pushes another 2 mph the total speed is the sum of the two rates.

Therefore, this choice is correct.

Please proceed to question 6 below.

.....

Question 6

A plane carries enough fuel to fly for t hours at p miles per hour with no wind. If it flies against a wind of y miles per hour for d miles, apply your knowledge to find how long it has taken.

(A) $\frac{d}{p + y}$

(B) $\frac{d}{p + y} + t$

(C) $\frac{d}{p - y} + t$

(D) $\frac{d}{p - y}$

172
1

In this problem, we use the formula:

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

Now the rate is a combined rate of normal speed plus or minus the wind which acts along the same path. The rate with the wind would be $(8 + w)$ and the rate against the wind would be $(8 - w)$.
Since the first "time" (going with the wind) is 2 less than the second "time," (going against the wind), our equation should state that:

first time = second time minus 2

Then we have:

$$\frac{m}{8 + 2} = \frac{m}{8 - 2} - 2$$

Therefore, this choice is correct.

Please proceed to question 10 below.

.....

172
2

Question 10

Apply your knowledge to find the solution of the equation:

$$\frac{15}{12 + x} = \frac{9}{12 - x}$$

Which domain contains the correct value of x ?

(A) $x > 8$

(B) $8 \geq x > 6.5$

(C) $6.5 \geq x > 3.5$

(D) $3.5 \geq x > 1.5$

If the rate in still water is 5 mph , while the rate of the stream is 1 mph , the rate downstream is 6 and the rate upstream is 4 . Then the time downstream is $\frac{20}{6}$ and the time upstream is $\frac{10}{4}$.

	R	x	T	=	D
Still	5				
Down	5 + 1		$\frac{20}{6}$		20
Up	5 - 1		$\frac{10}{4}$		10

$$\frac{20}{6} \neq \frac{10}{4}$$

Please return to page $\frac{177}{2}$ and try question 11 again.

.....

If we let c represent the rate of the current, we can organize our facts as follows:

	rate	x	time	=	distance
downstream	10 + c				12
upstream	10 - c				12

It is now possible to fill in the missing column, getting

$$\text{time downstream} = \frac{12}{10 + c}$$

$$\text{time upstream} = \frac{12}{10 - c}$$

Find the relationship between the two times given in the problem. You should now be able to write an equation and solve it. When you have completed the work and checked it, you will find that this choice is not correct.

Please return to page $\frac{181}{1}$ and try question 12 again.

174
1

The basic formula here is:

rate upstream = rate in still water - rate of current

Substituting the given quantities and using x for the rate in still water, we have the equation:

$p = x - m$ We solve for x and get

$p + m = x$

Then this choice is correct.

Please proceed to question 8 below.

.....

174
2

Question 8

A cyclist finds that he can travel d miles at x miles per hour, but that he can cover $d + 10$ miles in the same time if he increases his speed by 2 miles per hour. Apply your knowledge to find the equation which expresses the relationships in this problem.

(A) $\frac{d}{x} = \frac{d + 10}{2}$

(B) $\frac{d}{x} = \frac{d + 10}{x + 2}$

(C) $dx = (d + 10)(x + 2)$

(D) $\frac{d}{x} = \frac{d + 10}{x} + 2$

I The given information allows us to conclude that the rate downstream is 15 mph and the rate upstream is 9 mph. Then we can write:

	rate	x	Time	=	distance
downstream	15		$\frac{d}{15}$		d
upstream	9		$\frac{d}{9}$		d

Notice that we fill in the time column last.

II Now since the total time is 4, our equation is:

$$\frac{d}{15} + \frac{d}{9} = 4 \qquad \text{Multiplying by the LCD, 45}$$

$$3d + 5d = 180 \qquad [\ C$$

$$8d = 180 \qquad \div\ 8$$

$$d = 22\frac{1}{2} \text{ miles}$$

III CHECK

	R	x	T = D		
Down	15		$\frac{22.5}{15}$	22.5	$\frac{22.5}{15} + \frac{22.5}{9} \stackrel{?}{=} 4$
Up	9		$\frac{22.5}{9}$	22.5	$1.5 + 2.5 = 4$

Question 14

An airplane can cruise at 220 mph when there is no wind. It can fly 480 miles with the wind in the same time that it can fly 400 miles against the wind. Apply your knowledge to find the correct statement about w the rate of the wind in miles per hour.

(A) $w < 7$ (B) $7 < w < 14$

(C) $14 < w < 21$ (C) $w > 21$

176
1

Question 16

A plane takes 1 hour longer to fly 360 miles against a wind than it does to return. If the wind blows at 30 miles per hour, apply your knowledge to find the rate at which the plane flies in still air. Which statement about the rate, r , is correct?

(A) r is an integer less than 200

(B) r is an integer greater than 200

(C) r is smaller than 200 but it is not an integer

(D) r is larger than 200 but it is not an integer.

176
2

You seem to have some understanding of the problem, but you got an incorrect answer.

You are asked to find the time it takes to fly a certain distance d if the plane flies against a wind of y miles per hour when it normally flies at a rate of p miles an hour when there is no wind. The problem states that t is the maximum time it can fly at its normal rate in still air.

In what way does the quantity t affect the solution to this problem?

Please return to page 171/2 and try question 6 again.

You recall the method for solving fractional equations. Both sides of the equation must be multiplied by the least common multiple of the denominators (LCD) . Since the denominators cannot be factored, the LCD is equal to

$$(12 + x)(12 - x)$$

When we multiply both sides of the equation by the LCD, we get:

$$\begin{aligned} 15(12 - x) &= 9(12 + x) && [\ D \\ 180 - 15x &= 108 + 9x && \cancel{<} + 15x - 108 \\ 72 &= 24x && \cancel{<} \div 24 \\ 3 &= x \end{aligned}$$

Checking by substituting this value, we get:

$$\frac{15}{15} = \frac{9}{9}$$

Then this choice is correct since x has a value between 3.5 and 1.5 .

Please proceed to question 11 below.

Question 11

A boy can row 20 miles downstream in the same time that it takes him to row 10 miles upstream. If the current flows at 1 mile per hour, apply your knowledge to find which choice checks as the rate of rowing in still water.

(A) 2 mph

(B) 3 mph

(C) 4 mph

(D) 5 mph

$\frac{178}{1}$

If x represents the rate of the wind, we have:

	rate	·	time	=	distance
with wind	$100 + x$		$\frac{600}{100 + x}$		600
against wind	$100 - x$		$\frac{500}{100 - x}$		500

but we are told that one time is $\frac{4}{5}$ of the other. Therefore,

$$\frac{600}{100 + x} = \frac{4}{5}\left(\frac{500}{100 - x}\right) \quad \text{÷ 100}$$

$$\frac{6}{100 + x} = \frac{4}{5}\left(\frac{5}{100 - x}\right) \quad \text{[Reduce}$$

$$\frac{6}{100 + x} = \frac{4}{100 - x} \quad \text{LCD } (100 - x)(100 + x)$$

$$6(100 - x) = 4(100 + x) \quad \text{[D}$$

$$600 - 6x = 400 + 4x \quad \text{+6x, -400}$$

$$200 = 10x \quad \text{÷ 10}$$

$$20 = x$$

Then this choice is correct.

III CHECK:

	R	x	T	=	D
with wind	120		$\frac{600}{120}$		600
against wind	80		$\frac{500}{80}$		500

$$\frac{600}{120} = \frac{4}{5}\left(\frac{500}{80}\right)$$

$$5 = \frac{400}{80}$$

Proceed to question 16 on page $\frac{176}{1}$.

In problems of this nature, we are given a preliminary situation; a distance "d" covered during a time "x." We must recall the implied formula and then determine the time.

Organizing the facts in the two parts in chart form this would appear as follows:

	R	x	T	=	D
(1)	x		$\frac{d}{x}$		d
(2)	x + 2		$\frac{d + 10}{x + 2}$		d + 10

We then are presented with a second situation which is an adjustment of the first and we must re-apply the same rule. We must realize that the second speed is an <u>increase</u> of 2 miles per hour over the original; then the second speed is $x + 2$.

A relationship between the elements of the problem must be stated in order for you to write an equation.

Since time = $\frac{\text{distance}}{\text{rate}}$, your equation says that the two times involved are equal. That is exactly what the problem states; and, therefore, this choice is correct.

Please proceed to question 9 which follows.

.....

<u>Question 9</u>

A small blimp can travel 8 miles an hour if there is no wind. When the wind is blowing, it can cover m miles with the wind in 2 hours less time than it could cover the same m miles against the wind.

If w represents the rate of the wind in miles per hour, apply your knowledge to write an equation representing the facts in the problem.

(A) $\frac{m}{8 + w} = \frac{m}{8 - w}$

(B) $\frac{m}{8 + w} = \frac{m}{8 - w} + 2$

(C) $\frac{m}{8 + w} = \frac{m}{8 - w} - 2$

(D) $\frac{m + 2}{8 + w} = \frac{m}{8 - w}$

180
1

This problem merely asked you to check the values offered until you found the correct answer. With the rate in still water equal to 3 mph , and the rate of the current equal to 1 mph , the composition of these two produces the rates downstream and upstream which are respectively 4 and 2 .

	R	•	T	=	D
still	3				
down	3 + 1		$\frac{20}{4}$		20
up	3 - 1		$\frac{10}{2}$		10

Since these times are equal and that is the relationship noted in the problem, this choice is correct.

Note: the method for finding the value directly is outlined below.

I VARIABLES:

	R	•	T	=	D
down	x + 1		$\frac{20}{x + 1}$		20
up	x - 1		$\frac{10}{y - 1}$		10

II RELATIONSHIP: the times are:

$$\frac{20}{x + 1} = \frac{10}{x - 1} \quad \text{«(x + 1)(x - 1)}$$

$$20(x - 1) = 10(x + 1) \quad \text{[D}$$

$$20x - 20 = 10x + 10 \quad \text{«- 10x, + 20}$$

$$10x = 30 \quad \text{«÷ 10}$$

$$x = 3$$

Please proceed to question 12 on page 181 .
1

Question 12

The rate of a motorboat in still water is 10 mph . In $2\frac{1}{2}$ hours the boat traveled 12 miles upstream and returned. Apply your knowledge to find the set which contains the rate of the current.

(A) $\{4, 6, 9\}$

(B) $\{3, 5, 8\}$

(C) $\{1, 2, 7\}$

(D) None of these.

.....

I

Arranging our facts in a table and fitting in the time column last, we have:

	rate	• time	= distance
with wind	200 + w	$\frac{480}{220 + w}$	480
against wind	200 - w	$\frac{400}{220 - w}$	400

II

Then we can write an equation involving time as stated in the problem. Solving this equation gives us a value which does not agree with this choice.

Please return to page 175/2 and try question 14 again.

182/1

It was given that the boat's rate in still water is 12 mph and the rate of the current is 3 mph .

I

Using the given information, we find that the rate downstream is 15 mph and the rate upstream is 9 mph . If we let d represent the distance the boat can travel upstream, we have:

	rate	•	time	=	distance
downstream	15		$\frac{d}{15}$		d
upstream	9		$\frac{d}{9}$		d

II

The round trip up the stream and back down to the starting point is 4 hours. We can write an equation from this relationship. When we solve the equation, we should find that your value of d does not fit this statement.

Please return to page 167/1 and try question 13 again.

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182/2

Question 15

A plane can fly 100 miles an hour in still air. It flies 600 miles with the wind in $\frac{4}{5}$ of the time it would require to fly 500 miles against the same wind. Apply your knowledge to find the correct statement about the rate of the wind.

(A) It is less than 5 miles per hour.

(B) It is greater than 5 but less than 10 miles per hour.

(C) It is greater than 10 but less than 15 miles per hour.

(D) It is greater than 15 miles per hour.

I Good! Let's review the procedure.

Using our table, we have the following:

	rate	·	time	=	distance
with wind	$r + 30$		$\frac{360}{r + 30}$		360
against wind	$r - 30$		$\frac{360}{r - 30}$		360

II

Since the time against the wind is 1 more than time with the wind:

$$\frac{360}{r - 30} = \frac{360}{r + 30} + 1$$

Now we multiply by the LCD, $(r - 30)(r + 30)$

$$360(r + 30) = 360(r - 30) + (r - 30)(r + 30) \quad [D$$

$$360r + 10800 = 360r - 10800 + r^2 - 900 \quad [C$$

$$360r + 10800 = 360r - 11700 + r^2 \quad \checkmark + 11700, -360r$$

$$22500 = r^2$$

$$r = 150 \quad \text{and} \quad r = -150$$

Of course, -150 has no meaning in this situation. Then the rate is 150, and this choice is correct.

In order to check, we see that the rate with the wind is 180, which means it will take two hours with the wind. Since the rate against the wind is 120, it will take 3 hours against the wind.

	R	x	T	=	D
with	180		2		360
against	120		3		360

Then one time is one hour more than the other, which checks.

You have now finished this segment. Hand in the PUNCH CARD.

You should now be able to complete ASSIGNMENT 12, problems 16-20.

PROGRAMMED MATHEMATICS CONTINUUM
ALGEBRA - LEVEL ONE

ERRATA SHEET
VOLUME 12

ATTACH TO
BACK COVER

To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented. Certain typographical errors already exist and will also be corrected.

Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

PAGE	MISPRINT	CORRECTION	CHECK WITH CORRECTION MADE
45/2	Question 12	Question 1	
114/1	123/1	123/2	
125/2	...to page 150/1	...to page 150/2	